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Implication of risk adjusted discount rates on cycle stock and safety stock in a multi-period inventory model

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ABSTRACT

Inventory management is a major area of study in the field of production management, and applications often result in major efficiency gains and service level improvements. Economists believe that inventory levels play a pivotal role in the onset of economic recessions. Nevertheless, financial theories have given only cursory attention to inventory factors. The goal of this research is to apply tools of financial economics to address basic issues in inventory management. A capital asset pricing model is developed and applied to determine lot size and reorder points from minimizing present value of total cost subject to risk adjusted discount rates.

1. Introduction

The conventional approach to inventory-stocking decisions relies on a criterion that maximizes expected profits given a distribution of demand for the inventory item. The conventional approach is particularly unsatisfying when considering that financial economists approach other corporate finance decisions with models that account for risk in an economically meaningful way, but do not do so when modeling inventory decisions. This is puzzling since these critical decisions should be made according to the same sound financial principles that guide decision making in the rest of the organization. The contingent-claims approach presented here maps the demand and payoffs from an inventory policy onto the price of an underlying state variable, builds a portfolio of options that replicates the payoffs, and then derives the value of an inventory policy using option pricing models in which the value of the policy is contingent on the financial characteristics of the underlying asset.

Standard procedures for evaluating future cash flows are to find an appropriate discount rate consistent with the cash flow's risk and then to derive a present value. The fundamental assumption in these discounted cash flow (DCF) models is that a discount rate exists for each cash flow that can yield an equivalent present value of the cash flow. Several assumptions are necessary in using these evaluations for decision making such as independence of alternatives, a perfect market allowing unlimited borrowing and lending at a risk-free rate, and usually some form of the capital asset pricing model (CAPM) to obtain a rate that is based only on the cash flow's contribution to overall market risk. This last assumption has both market assumptions as well as an assumption about the ability to determine the risk of the cash flow before making the decision. This is the critical aspect to be considered in the present study. In this paper the total cost function for inventory management of a company is modeled with risk adjusted cost to explicitly introduce risk in the inventory

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management policies. The risk adjustment reflects uncertainties about the reassessment through time of the expected cash flow with the return market, which for practical applications has often been interpreted to be the covariance of the company's return with the market return.

The risk of the cash flow is determined only after a decision is made because the decision determines the sales level and its standard deviation. With a low capacity for sales, sales are often low regardless of the demand. The standard deviation of sales in that case would be low. For high capacity, sales would be equivalent to demand and, hence, have a higher standard deviation. The risk as measured by standard deviation is then determined by the capacity or production decision. Therefore, decisions must be modeled by evaluating future cash flows. We propose to incorporate this information by using the capital asset pricing model (CAPM) directly. An alternative could be to construct a utility function directly, but such a computation would be tedious due to the difficulty in assessing the utility of all stakeholders.

Investments in inventory represent a significant portion of most the firms' total investments. Given the significant level of investments in inventory, it is surprising that most of the models in the management science literature do not analyze inventory decisions in the framework commonly used to analyze investments in fixed assets. In most of the firms, procedures for analyzing investments in fixed assets are oriented towards determining the present value of investments by appropriately discounting the relevant cash flows for the timing and the risk of the cash flows. More often than not, the effect of inventory decisions on the timing and risk of cash flows is ignored.

Early efforts include the work of Hadley (1964), Beranek (1967), and Rummel (1985). These papers have focused on the issue of the timing of the cash flow in deterministic models, while assuming that the opportunity cost of capital is known and independent of inventory decisions. This is appropriate for deterministic models, but the effect of inventory decisions on the risk of cash flows is an important issue to consider when extending this analysis to stochastic inventory models. The existing stochastic inventory models incorporate the risk of holding inventory by specifying the opportunity cost of capital. Our research is based on the work of Sharpe (1964) and Lintner (1965) on the capital asset pricing model to value the cash flows.

The concept of discounting risky cash flows using a risk adjusted discount rate originated in the finance discipline. Fama (1977) is a key paper establishing the point of view that the present value of future cash flows is the current expected value of the flows discounted at a risk adjusted interest rate. The risk adjustment is based on CAPM. This approach has been particularly influential in capital budgeting applications.

In a reassessment of the earlier work on risk adjusted discount rates, Fama and French (1997) argue that there are a number of flaws with traditional risk adjusted discount rates and propose models which seem to offer improvement.

Meyers and Cohn (1987) popularized the application of risk adjusted discount rates based on CAPM to the pricing of insurance policies.

Kozik (1994) considers rate making methods based on CAPM, critiques the methodologies used in other students to estimate underwriting betas for application in CAPM, and argues that reliable estimates of underwriting betas do not exist.

Another approach introduced by researchers in finance uses option pricing theory (Smith and Nau, 1995). D'Arcy and Garven (1990) compare option pricing rate making results to those of a number of other alternatives, including CAPM based approaches. D'Arcy and Garvin's tests indicate that this approach compares favorably with other pricing approaches, and in general, does better than those based on CAPM.

According to Cummings (1985), the approach attempts to price so that the present value of the equity of the insurance company is equal to a call option on excess of the company's assets over its liabilities.

Butsic (1988) introduced the use of risk adjusted rates to discount liabilities. The paper argues for the use of risk-adjusted discount rates when discounting liabilities. Since CAPM when applied to assets has a positive risk adjustment, it is reasonable that a model for discounting liabilities should have a negative risk adjustment. Although Butsic's risk adjustment is conceptually similar to a CAPM risk adjustment, Butsic actually used industry profitability data published in *Best's Aggregates and Averages* to derive his risk adjustment, rather than measuring covariances of losses with market returns.

There are numerous recent investigations of risk within the context of the financial management of inventory problems. Borgonovo and Peccati (2009) consider the effect of risk characterization on the determination of inventory policies. Bulinskaya (2003) develops a discrete-time model of inventory control using investment policy while Abuo-El-Ata et al. (2003) offer a geometric programming approach. Kelle and Miller (2001) investigate how the possibility of splitting orders among vendors impacts stock-out risk. Bylka and Rempala (2001) provide a multi-product allocation problem to minimize the sum of production and inventory holding costs over a finite horizon.

Following the review of risk adjusted discount rates, the next section shows how risk considerations affect the cycle stock (lot size) and safety stock (reorder point) when both these decision are made simultaneously in a multi-period setting. Although there are many inventory models that can be used to capture the interrelationship between the cycle stock and safety stock, in this project we use the (Q, r) inventory model with setup, inventory, and back-order costs. This model is then analyzed under the objective of minimizing the present value of total cost, using the CAPM to value the uncertain cash flows. Accordingly, in the model the risk of demand is measured by its covariance with the stock market return.

2. Notation and model development

This section introduces the multi-period approximation to the (Q, r) model and identifies the conditions under which minimizing the average annual cost is equivalent to minimizing the present value of total costs. The following notation is used.