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The effects of lumpy demand and shipment size constraint: A response to "Revisit the note on supply chain integration in vendor-managed inventory"

Boray Huang *, Zhisheng Ye

Department of Industrial and Systems Engineering, National University of Singapore, 117576, Singapore, Singapore

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ABSTRACT

This paper responds to a comment by Wang et al. [3] regarding the disagreement between Yao et al. [4] and van der Vlist et al. [2] on the impact of vender-managed-inventory (VMI). We explore the factors which affect the shipment size from the vendor to the buyer and identify the conditions where the shipment size will increase/decrease under VMI. A numerical example also shows when and how the inventory shifts between the supplier and the buyer.

1. Introduction

Wang et al. [3] try to resolve a disagreement between Yao et al. [4] and van der Vlist et al. [2] regarding the impact of vendor-managed-inventory (VMI) on a supply chain. Yao et al. [4] use a two-tier EOQ model to investigate the costs and order behaviours of the supply chain members in both VMI and non-VMI scenarios. They find some interesting analytical results and identify the distribution of the VMI benefit between the supplier and the buyer. van der Vlist et al. [2] argue that Yao et al. overstate the inventory needed at the supplier, and extend Yao et al.'s model to incorporate shipping costs. The findings in Yao et al. [4] and van der Vlist et al. [2] are different in many ways. For example, Yao et al. assert that the buyer's order size shall decrease under VMI, but van der Vlist et al. have an opposite result. In their response [5], Yao et al. argue that van der Vlist et al. [2] make problematic assumptions on the shipping costs. Wang et al. [3] also support Yao et al.'s original results by studying a special case.

It is interesting to note that, while the arguments of Yao et al. [5] and Wang et al. [3] focus on the validity of van der Vlist et al.'s cost assumptions, some of the results in van der Vlist et al. do not rely on these assumptions. To clarify the causes of the disagreement, we relax all the extra assumptions in van der Vlist et al. [2], let the delivery cost be *zero* and revisit the original paper of Yao et al. [4]. In this note we discuss two important factors on the benefit of VMI: The lumpy demand at the suppliers and the shipment size constraint. Both are neglected in all these papers. Our study clearly shows how these two factors affect the optimal order quantities, and provides intuitive insights for the implementation of VMI.

2. The cost functions and the optimal solutions

We use the same models and notations in Yao et al. [4]. The supply chain consists of one buyer and one supplier. The buyer's order size is q and the supplier's order size is Q. From Crowston et al. [1] and Zipkin ([6], Theorem 5.3.2), it is obvious that the supplier's order pattern in van der Vlist et al.'s synchronized case is the optimal stationary pattern for both the supplier and the entire supply chain. In other words, the supplier's ordering pattern presumed in Yao et al. [4], which always keeps excessive q units on hand, is an inferior one.

With the order pattern in the synchronized case of van der Vlist et al., the total cost function for the entire system in the non-VMI (NV) case is

$$TC_{\rm NV}(Q,q) = TC^{\rm S}_{\rm NV}(Q,q) + TC^{\rm R}_{\rm NV}(q) \tag{1}$$

where

$$TC^{\rm S}_{\rm NV}(Q,q) = \frac{Cr}{Q} + H\left(\frac{Q-q}{2}\right) \tag{2}$$

$$= \left[\frac{Cr}{Q} + H\left(\frac{Q}{2}\right)\right] - H\left(\frac{q}{2}\right) \tag{3}$$

$$TC^{\rm R}_{\rm NV}(q) = \frac{cr}{q} + h\left(\frac{q}{2}\right) \tag{4}$$

$Q \geq q > 0$.

TC^S_{NV} and TC^R_{NV} are respectively the cost functions of the supplier and of the buyer. Our cost function TC_{NV} is different from the ones in [2] and [4] for the non-VMI case (e.g., Eq. (1) of [2] when $T=0$ and Eq. (3) of [4].) The difference is due to the fact that both [2] and [4] overlook the effect of lumpy demand (i.e., the last term in (3)) faced

* Corresponding author.
E-mail address: isehb@nus.edu.sg (B. Huang).

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by the supplier. Interestingly, this effect is included in the supplier's cost functions of their VMI models.

[1]Note that the lumpy demand from the buyer does not affect the supplier's optimal decision of the order size Q unless the buyer's order size is sufficiently large (that is, when the condition $Q \geq q$ is concerned.) However, it *does* affect the supplier's cost performance. Thus the effect of lumpy demand at the supplier becomes an important factor for the decision of the shipment size in the VMI case.

Go back to the non-VMI (NV) case, the optimal order quantities can be determined directly from the classic EOQ solution to (3) and (4). That is,

$$q^* = \sqrt{\frac{2cr}{h}} \tag{5}$$

$$Q^* = \sqrt{\frac{2Cr}{H}}. \tag{6}$$

When $q^* > Q^*$, the supplier's order size will be raised to q^* because, as shown in the proof of Proposition **1**, it is never optimal for the supplier to have an order size $Q < q$. As a result, we have the optimal order quantities and the optimal cost functions for the non-VMI case: (All the proofs in this note are shown in Appendix.)

Proposition 1. *In the non-VMI (NV) case, the optimal order quantities Q^*_{NV} and q^*_{NV} under the condition $q^*_{NV} \leq Q^*_{NV}$ are*

$$q^*_{NV} = \sqrt{\frac{2cr}{h}}$$

$$Q^*_{NV} = \max\left\{q^*_{NV}, \sqrt{\frac{2Cr}{H}}\right\}$$

The optimal cost function of the entire system without VMI is

$$TC^*_{NV} = \begin{cases} \sqrt{2crh} + \sqrt{2CrH} - H\sqrt{\frac{cr}{2h}} & ;\text{when } c \leq \frac{h}{H}C \\ \left(\frac{C}{2c} + 1\right)\sqrt{2crh} & ;\text{when } c > \frac{h}{H}C. \end{cases} \tag{7}$$

When VMI is implemented, the buyer's order cost c is replaced by c', where $c' \leq c$ as assumed in Yao et al. [4]. The total cost function (1) can therefore be rewritten as

$$TC_{VMI}(Q,q) = TC^S_{VMI}(Q,q) + TC^R_{VMI}(q) \tag{8}$$

$$= \left\{\frac{Cr}{Q} + H\left(\frac{Q-q}{2}\right)\right\} + \left\{\frac{c'r}{q} + h\left(\frac{q}{2}\right)\right\} \tag{9}$$

$$= \left[\frac{Cr}{Q} + H\left(\frac{Q}{2}\right)\right] + \left[\frac{c'r}{q} + (h-H)\left(\frac{q}{2}\right)\right] \tag{10}$$

where $Q \geq q > 0$. We define TC^S_{VMI} and TC^R_{VMI} as the total costs at the supplier's side and at the buyer's side, respectively. Within the first square bracket of (10) we have a classic EOQ cost which depends only on Q. The second square bracket of (10) also contains a classic EOQ cost but depends only on q. When the decision making is centralized in the VMI case, the optimal order quantities and the optimal cost can be obtained by optimizing the two EOQ costs in the square brackets of (10) with respect to Q and q separately. We can immediately obtain

$$Q^* = \sqrt{\frac{2Cr}{H}} \tag{11}$$

$$q^* = \sqrt{\frac{2c'r}{h-H}} \tag{12}$$

when $c' \leq \frac{h-H}{H}C$. A tricky case occurs when $c' > \frac{h-H}{H}C$, which is equivalent to $q^* > Q^*$. van der Vlist et al. [2] indicates that we can let $q^*_{VMI} = Q^*_{VMI}$ whenever $h \leq H$, but they do not solve the problem of the optimal order quantities in the more general situation of $\{q^* > Q^*\}$, nor do they provide exact expressions of $\{Q^*_{VMI}, q^*_{VMI}\}$ when $h \leq H$. We find $\left\{Q = q = \sqrt{\frac{2Cr}{H}}\right\}$ is not an optimal solution to TC_{VMI} whenever $c' > \frac{h-H}{H}C$ (including the case of $h \leq H$.) In fact, we have the following proposition:

Proposition 2. *In the VMI case, the optimal order quantities Q^*_{VMI} and q^*_{VMI} under the constraint $Q \geq q$ are*

$$(Q^*_{VMI}, q^*_{VMI}) = \begin{cases} \left(\sqrt{\frac{2Cr}{H}}, \sqrt{\frac{2c'r}{h-H}}\right) & ;\text{when } c' \leq \frac{h-H}{H}C \\ \left(\sqrt{\frac{2(c'+C)r}{h}}, \sqrt{\frac{2(c'+C)r}{h}}\right) & ;\text{when } c' > \frac{h-H}{H}C. \end{cases}$$

The optimal cost function under VMIis

$$TC^*_{VMI} = \begin{cases} \sqrt{2c'r(h-H)} + \sqrt{2CrH} & ;\text{when } c' \leq \frac{h-H}{H}C \\ \sqrt{2(c'+C)rh} & ;\text{when } c' > \frac{h-H}{H}C. \end{cases} \tag{13}$$

3. The impact of VMI

The supplier's cost function TC^S provides a quick answer to the impact of lumpy demand. The last term of (3) is independent of Q and represents the impact of the lumpy demand in the non-VMI case. The total cost function of the VMI case has the same structure. It is then easy to obtain the following property which holds in both VMI and non-VMI cases.

Corollary 1. *The cost at the supplier's side is non-increasing in the buyer's order size.*

According to Corollary 1, the centralized decision maker in the VMI case may want to increase the buyer's order size from its non-VMI solution ($\sqrt{2c'r/h}$) in order to benefit from the cost reduction at the supplier. In fact, the buyer's order size shall be set to a point where the marginal increase of the buyer's cost equals to the marginal saving of the supplier's cost if the $q \leq Q$ constraint is not concerned. That is, q should satisfy

$$\frac{d}{dq}TC^R_{VMI} = -\frac{c'r}{q^2} + \frac{h}{2} = -\left(-\frac{H}{2}\right) = -\frac{\partial}{\partial q}TC^S_{VMI} \tag{14}$$

[1] Strictly speaking, the buyer's cost function TC^R_{NV} should also be modified if the end demand is discrete. Such modification, however, will not affect our main results.