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The viewpoint on "Optimal inventory policy with non-instantaneous receipt under trade credit by Ouyang, Teng, Chuang and Chuang"

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ABSTRACT

Ouyang et al. [2005. International Journal of Production Economics 98, 290–300] develop an inventory model with non-instantaneous receipt under trade credit, in which the supplier provides not only a permissible delay but also a cash discount to the retailer. They establish a criterion to find the optimal order cycle such that the total relevant cost per unit time is minimized. Although their inventory model is correct and interesting, their solution procedure has shortcomings such that it cannot locate all optimal order cycles. So, the main purpose of this paper will remove the shortcoming of Ouyang et al. (2005) and present a solution procedure to search for the entirely optimal order cycles. Furthermore, this paper reveals an example to show that Ouyang et al.'s (2005) solution procedure does not work for locating the optimal order cycle to that example, however, our solution procedure does. In sum, this paper improves Ouyang et al. (2005).

1. Introduction

The traditional economic order quantity model assumes that an entire order is received into the inventory at one time (i.e. infinite replenishment rate). In empirical observations, the order quantity is frequently received gradually over time and the inventory level is depleted at the same time it is being replenished. Ouyang et al. (2005) think that this version of the economic order quantity (EOQ) model is known as the non-instantaneous receipt (i.e. finite replenishment rate) model. The "non-instantaneous receipt" and economic production quantity (EPQ) are the same in the mathematical model. However, there is a slight different in managerial implication. The "instantaneous receipt" means that the retailer receives the order quantity and sales at the same time in the retailer business. The EPQ means that the manufacturer produces the products and sales at the same time in the manufacturing industry. The concept of "instantaneous receipt" can be observed in Jaber et al. (2008), Liao (2008), Ouyang et al. (2008) and Yoo et al. (2009).

The inventory problem consists of two parts: (1) the modeling, and (2) the solution procedure. The modeling can provide insight to solve the inventory problem and the solution procedure involves the implementation of the inventory model. From the viewpoint of practice, the modeling and the solution procedure are equally important. Ouyang et al. (2005) develop an inventory model with non-instantaneous receipt under trade credit, in which the supplier provides not only a permissible delay but also a cash discount to the retailer. They establish a criterion to find the optimal order cycle so that the total relevant cost per unit time is minimized. Although their inventory model is correct and interesting, their solution procedure has shortcomings such that it cannot locate all optimal order cycles. So, the main purpose of this paper will remove the shortcoming of Ouyang et al. (2005) and present a solution procedure to search for the entirely optimal order cycles. Furthermore, this paper reveals an example to show that Ouyang et al. (2005) solution procedure does not work for locating the optimal order cycle to that example, however, our solution procedure does. In sum, this paper improves Ouyang et al. (2005).

2. Model formulation

Adopting the same assumptions and notation as in Ouyang et al. (2005), the retailer has two policies (Policies (I) and (II)) to be decided.

Policy (I): the retailer accepts a cash discount and makes payment at M_1, and Policy (II): the retailer does not accept a cash discount and makes payment at M_2.

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So, the total relevant cost per unit time can be divided into two cases to be discussed.

Case 1: The retailer adopts Policy (I)

In this case, Ouyang et al. (2005) show that

$$\overline{TC}_1(T)=\begin{cases} TC_2(T) & \text{if } 0<T\le M_1, & (1a)\\ TC_1(T) & \text{if } M_1\le T\le \dfrac{KM_1}{D}, & (1b)\end{cases}$$

where

$$TC_2(T)=\frac{S}{T}+\frac{DTh\rho}{2}+c(1-r)D-pI_d\left[\frac{DT^2}{2}+DT(M_1-T)\right]/T, \quad (2)$$

and

$$TC_1(T)=\frac{S}{T}+\frac{DTh\rho}{2}+c(1-r)D+cI_c(1-r)\left[\frac{D(T-M_1)^2}{2}\right]/T - \frac{pI_dDM_1^2}{2T}. \quad (3)$$

Case 2: The retailer adopts Policy (II)

In this case, Ouyang et al. (2005) show that

$$\overline{TC}_2(T)=\begin{cases} TC_4(T) & \text{if } 0<T\le M_2, & (4a)\\ TC_3(T) & \text{if } M_2\le T\le \dfrac{KM_2}{D}, & (4b)\end{cases}$$

where

$$TC_4(T)=\frac{S}{T}+\frac{DTh\rho}{2}+cD-pI_d\left[\frac{DT^2}{2}+DT(M_2-T)\right]/T, \quad (5)$$

and

$$TC_3(T)=\frac{S}{T}+\frac{DTh\rho}{2}+cD+cI_c\left[\frac{D(T-M_2)^2}{2}\right]/T-pI_d\left(\frac{DM_2^2}{2}\right)/T. \quad (6)$$

Combining Cases 1 and 2, the total relevant cost per unit time can be expressed as follows:

$$TC(T)=\begin{cases} \overline{TC}_1(T) & \text{if the retailer adopts Policy (I)}, & (7a)\\ \overline{TC}_2(T) & \text{if the retailer adopts Policy (II)}. & (7b)\end{cases}$$

3. The convexity of $TC_i(T)$ (i=1, 2, 3, 4)

For convenience, we treat all $TC_i(T)$ (i=1, 2, 3, 4) are defined on $T>0$. Eqs. (2), (3), (5) and (6) yield that

$$TC_1'(T)=-\left\{\frac{2S+DM_1^2[cI_c(1-r)-pI_d]}{2T^2}\right\}+\frac{D[h\rho+cI_c(1-r)]}{2}, \quad (8)$$

$$TC_1''(T)=\frac{2S+DM_1^2[cI_c(1-r)-pI_d]}{T^3}, \quad (9)$$

$$TC_2'(T)=\frac{-S}{T^2}+\frac{D(h\rho+pI_d)}{2}, \quad (10)$$

$$TC_2''(T)=\frac{2S}{T^3}>0, \quad (11)$$

$$TC_3'(T)=-\left\{\frac{2S+DM_2^2[cI_c-pI_d]}{2T^2}\right\}+\frac{D(h\rho+cI_c)}{2}, \quad (12)$$

$$TC_3''(T)=\frac{2S+DM_2^2(cI_c-pI_d)}{T^3}, \quad (13)$$

$$TC_4'(T)=\frac{-2S}{T^2}+\frac{D(h\rho+pI_d)}{2}, \quad (14)$$

and

$$TC_4''(T)=\frac{2S}{T^3}>0. \quad (15)$$

Let

$$H_1=2S+DM_1^2[cI_c(1-r)-pI_d], \quad (16)$$

and

$$H_2=2S+DM_2^2[cI_c-pI_d]. \quad (17)$$

Eqs. (9), (11), (13) and (15) imply the following results.

Lemma 1. *(i) $TC_1(T)$ is convex on $T>0$ if $H_1>0$. Furthermore, $TC_1'(T)>0$ and $TC_1(T)$ is increasing on $T>0$ if $H_1\le 0$.*

(ii) $TC_2(T)$ is convex on $T>0$.

(iii) $TC_3(T)$ is convex on $T>0$ if $H_2>0$. Furthermore, $TC_3'(T)>0$ and $TC_3(T)$ is increasing on $T>0$ if $H_2\le 0$.

(iv) $TC_4(T)$ is convex on $T>0$.

Letting

$$TC_i'(T)=0, (i=1,2,3,4) \quad (18)$$

and solving Eq. (18), we obtain that

$$T_1=\sqrt{\frac{2S+DM_1^2[cI_c(1-r)-pI_d]}{D[h\rho+cI_c(1-r)]}} \quad \text{if } H_1>0, \quad (19)$$

$$T_2=\sqrt{\frac{2S}{D(h\rho+pI_d)}}, \quad (20)$$

$$T_3=\sqrt{\frac{2S+DM_2^2[cI_c-pI_d]}{D(h\rho+cI_c)}} \quad \text{if } H_2>0, \quad (21)$$

and

$$T_4=\sqrt{\frac{2S}{D(h\rho+pI_d)}} \quad (22)$$

are the respective solutions of Eq. (18).

If T_i exists, then $TC_i(T)$ is convex. So, we have

$$TC_i'(T)\begin{cases} <0 & \text{if } 0<T<T_i, (23a)\\ =0 & \text{if } T=T_i, (23b)\\ >0 & \text{if } T>T_i. (23c)\end{cases}$$

Eqs. (23)a–c imply that $TC_i(T)$ is decreasing on $(0,T_i]$ and increasing on $[T_i,\infty)$ for i=1, 2, 3, 4.

4. Meanings of Δ_i $(i=1,2,3,4)$

Case 1: The retailer adopts Policy (I)

In this case, we have

$$\overline{TC}_1(T)=\begin{cases} TC_2(T) & \text{if } 0<T\le M_1, & (1a)\\ TC_1(T) & \text{if } M_1\le T\le \dfrac{KM_1}{D}. & (1b)\end{cases}$$

Then, we find $TC_1(M_1)=TC_2(M_2)$. Hence $\overline{TC}_1(T)$ is continuous and well-defined on $T>0$. Furthermore, we have

$$TC_1'(M_1)=TC_2'(M_1)=\frac{-2S+DM_1^2(h\rho+pI_d)}{2M_1^2}, \quad (24)$$

and

$$TC_1'\left(\frac{KM_1}{D}\right)=\frac{-2S+\frac{M_1^2}{D}[cI_c(1-r)(K^2-D^2)+pI_dD^2+hK(K-D)]}{2\left(\frac{KM_1}{D}\right)^2}. \quad (25)$$