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Note on: A necessary and sufficient conditions for the existence of the optimal solution of a single-vendor single-buyer integrated production-inventory model with process unreliability consideration

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ABSTRACT

Chung (2008) discussed the necessary and sufficient conditions for the existence of the optimal solution of an integrated production-inventory model developed by Huang (2004), which allowed a vendor and a buyer to minimize their expected integrated total cost function with an imperfect production process. The objective of Chung (2008) was to improve the solution procedure presented in Huang (2004). In this study, we identify three errors, irrelevant conditions, and inappropriate examples used in Chung (2008). We provide a reformulation of the model and the correct necessary and sufficient conditions for optimality.

1. Introduction

Chung (2008) recently claimed that the solution procedure provided in Huang (2004) was likely to cause misunderstandings. The objective of Chung (2008) was to improve the solution procedure in Huang (2004) by providing the necessary and sufficient conditions for the existence of the optimal solution. In this note, we identify three errors, irrelevant conditions for optimality, and inappropriate examples used in Chung (2008). We reformulate the expected integrated total cost function and provide the correct necessary and sufficient conditions for optimality by using the conventional optimization method.

In reformulating the model, we use the following notations:

Q	lot size per production run
D	annual demand
P	production rate, $P>D$
S_V	setup cost per production run for the vendor
S_B	cost of placing an order for the buyer
h_V	unit stock-holding cost per item per year for the vendor
h_B	unit stock-holding cost per item per year for the buyer
n	number of shipments per lot from the vendor to the buyer
T	time interval between successive delivery
F	transportation cost per shipment
Y	percentage of defective items, a random variable
$f(y)$	probability density function of Y
v	the vendor's unit warranty cost of defective items
x	screening rate
d	unit screening cost
$EK(Q, n)$	expected annual integrated total cost

2. Errors in Chung (2008)

The first error involves the incorrect treatment of the random variable Y with a known probability density function, which represents percentage of defective items in a production lot. The error originated from Huang (2004). When Huang (2004, p. 95) expressed the vendor's holding cost, the deterministic order cycle length T was replaced with $(1-Y)Q/(nD)$, which follows a known probability distribution. As is self-evident, the two notions are substantially different, and they cannot be equated. All of the follow-up equations including the expected integrated total cost function (Eq. (5) in Huang, 2004, p. 95) were also subjected to the same error. Without correcting such an error, Chung (2008) used Huang's incorrect expected integrated total cost function (Eq. (5) in Huang, 2004, p. 95) for his analyses. The second error is that Chung (2008, p. 271) provided two necessary conditions for optimality (Eq. (7a–c) and Eq. (15a–c)), but did not provide any sufficient condition for optimality of the expected integrated total cost function, unlike he claimed in the purpose of paper. The third error also originated from Huang (2004). Assumption (7) in Chung

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(2008, p. 270) should be corrected to “The vendor pays the buyer for the warranty of items of poor quality as a single payment at the end of the buyer’s 100% screening process.”

3. Irrelevant conditions and inappropriate examples used in Chung (2008)

Chung (2008) considered the following three vendor’s production requirement conditions for optimization of n^*: $1-(DM/P)>0$, $1-(DM/P)=0$, and $1-(DM/P)<0$. The first condition implies that the vendor’s production must be set to larger than the sum of the buyer’s demand and the expected number of defective items. Thus, the buyer-vendor coordination is feasible. The second and third conditions, however, imply that the production requirement is set to equal or less than the sum of the buyer’s demand and the expected number of defective items. Consequently, there will be a high chance of shortages to occur, and the buyer-vendor coordination is unlikely to be feasible. Thus, these conditions are naturally irrelevant to the buyer-vendor coordination. As such, they have not been considered by Huang (2004) and other studies in formulating their integrated models.

Chung (2008) also presented three examples in support of the first and third conditions for n^*. In Example 1, where $1-(DM/P)>0$, Chung (2008) used the same parameter values used in Huang (2004) and obtained the same optimization results. In Example 2, keeping all other parameter values the same as in Example 1, Chung (2008) changed the holding costs of the buyer and vendor from h_B=\$5/unit/year and h_V=\$2/unit/year to h_B=\$2/unit/year and h_V=\$6/unit/year. The new holding costs are simply inappropriate for the buyer-vendor coordination. When the buyer observes the vendor’s holding cost of \$6/unit/year, the buyer is unlikely to share the vendor’s high holding cost through their coordination. In Example 3, Chung (2008) changed the annual production rate from P=160,000 units/year to P=51,000 units/year, keeping all other parameters the same as in Example 1. As a result, the third condition became negative, and the buyer-vendor coordination became infeasible. As such, Examples 2 and 3 are inappropriate to the buyer-vendor coordination.

Chung (2008) also used two additional conditions for optimization: $\Gamma(n)$ for $Q^*(n)$ and G for n^*. They are defined as parts of $\partial EK(Q,n)/\partial Q$ and are the same under the second condition: $1-(DM/P)=0$. $\Gamma(n)$ must be positive to find $Q^*(n)$. Otherwise, the optimal solution cannot be obtained. With both $1-(DM/P)>0$ and $G\geq 0$, Chung (2008) was able to obtain the same optimization results as Huang (2004). On the other hand, when $G<0$, the optimal solution becomes ($Q^*(n^*=1)$, $n^*=1$); this implies that the vendor and the buyer will simply rely on the cost-inefficient single-setup-single-delivery policy. As with the first condition, $1-(DM/P)>0$, the signs of $\Gamma(n)$ and G are important for Chung’s (2008) solution procedure, but no economically meaningful explanations are provided for these conditions. Given the number of parameters involved, however, it is unlikely that these conditions can provide practitioners with any meaningful guidelines for optimization.

It seems that Chung (2008) biased his work to achieve mathematical results only, without contemplating the main purpose of Huang (2004). As such, the results in Chung (2008) do not shed any new insights on either the solution procedure or the main purpose of Huang (2004). Furthermore, it is important to note that the optimization procedure used in both Huang (2004) and Chung (2008) is inefficient compared to the conventional optimization method.

4. Reformulation and optimization

In this section, we reformulate the integrated production-inventory model to incorporate the previously stated correction. In order to overcome the first error and later obtain the expected value of the integrated buyer-vendor total cost, it would be more reasonable to use the expected value of the percentage of defective items contained in each lot from the start of model development. For our reformulation, we use the contract quantity Q and the number of deliveries n as our decision variables. We assume that the expected number of good quality items in each lot size $(1-E(Y))Q/n$ delivered from the vendor is equal to the demand during the order cycle T. The rationale for this is that the demand during the order cycle T can on average be met by the expected number of good quality items of each lot size Q/n with the expected percentage of $(1-E(Y))$.

Adjusting all the relevant cost terms of both the vendor’s and the buyer’s total cost functions in Huang (2004, p. 95) with $DT=(1-E(Y))Q/n$ and adding the resulting cost functions, we obtain the expected integrated total cost function for the buyer and vendor as follows:

$$EK(Q,n)=\frac{D(S_B+S_V+nF)}{Q(1-E(Y))}+\frac{(d+vE(Y))D}{(1-E(Y))}+\frac{Q}{2n}\left[\left\{(1-E(Y))+\frac{2DE(Y)}{x(1-E(Y))}\right\}h_B+\left\{\frac{(2-n)D}{P(1-E(Y))}+n-1\right\}h_V\right]. \tag{1}$$

Taking the first derivatives of Eq. (2) with respect to n and Q, setting them equal to zero (i.e., necessary conditions for optimality), and solving for n and Q simultaneously, we obtain the following formulas:

$$n^*=\sqrt{\frac{(S_B+S_V)\left[\left\{(1-E(Y))+\frac{2DE(Y)}{x(1-E(Y))}\right\}h_B+\left\{\frac{2D}{P(1-E(Y))}-1\right\}h_V\right]}{\left\{1-\frac{D}{P(1-E(Y))}\right\}Fh_V}}$$

and

$$Q^*=\sqrt{\frac{2nD(S_B+S_V+nF)}{\left[\left\{(1-E(Y))+\frac{2DE(Y)}{x(1-E(Y))}\right\}h_B+\left\{\frac{(2-n)D}{P(1-E(Y))}+n-1\right\}h_V\right](1-E(Y))}}. \tag{2}$$

If the optimal number of deliveries n^* in Eq. (2) is not an integer, then we choose n which yields $\min\{EK(Q^*,n^-),EK(Q^*,n^+)\}$ in Eq. (1), where n^+ and n^- represent the nearest integers larger and smaller than n^*. Note that Eq. (2) indicates that $P>D/(1-E(Y))$. However, in order to prevent shortages from occurring, P must be greater than $D/(1-\max(Y))$, where $\max(Y)$ indicates the largest outcome in the sample space of Y. As shown in Appendix A, the Hessian matrix of Eq. (1) is positive definite. This ensures that the sufficient condition for optimality is met, and the expected integrated total cost function $EK(Q, n)$ is jointly convex at the optimal solution (Q^*,n^*).

Appendix A. Sufficient condition for optimality

Since the expected integrated total cost function $EK(Q, n)$ is a function of two decision variables Q and n, we define the Hessian