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Computing the non-stationary replenishment cycle inventory policy under stochastic supplier lead-times

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ABSTRACT

In this paper we address the general multi-period production/inventory problem with non-stationary stochastic demand and supplier lead-time under service level constraints. A replenishment cycle policy (R^n,S^n) is modeled, where R^n is the nth replenishment cycle length and S^n is the respective order-up-to-level. We propose a stochastic constraint programming approach for computing the optimal policy parameters. In order to do so, a dedicated global chance-constraint and the respective filtering algorithm that enforce the required service level are presented. Our numerical examples show that a stochastic supplier lead-time significantly affects policy parameters with respect to the case in which the lead-time is assumed to be deterministic or absent.

1. Introduction

An interesting class of production/inventory control problems is the one that considers the single location, single product case under stochastic demand. One of the well-known policies that can be adopted to control such a system is the "replenishment cycle policy' (R,S). Under the non-stationary demand assumption this policy takes the form (R^n,S^n), where R^n denotes the length of the nth replenishment cycle, and S^n the order-up-to-level value for the nth replenishment. This easy to implement inventory control policy yields at most $2N$ policy parameters fixed at the beginning of an N-period planning horizon. For a discussion on inventory control policies see Silver et al. (1998). The replenishment cycle policy provides an effective means of damping the planning instability. Furthermore, it is particularly appealing when items are ordered from the same supplier or require resource sharing. In such a case all items in a coordinated group can be given the same replenishment period. Periodic review also allows a reasonable prediction of the level of the workload on the staff involved and is particularly suitable for advanced planning environments. For these reasons, as stated by Silver et al. (1998), (R,S) is a popular inventory policy. Due to its combinatorial nature, the computation of (R^n,S^n) policy parameters is known to be a difficult problem to solve to optimality. An early approach proposed by Bookbinder and Tan (1988) is based on a two-step heuristic method. Tarim and Kingsman (2004, 2006) and Tempelmeier (2007) propose a mathematical programming approach to compute policy parameters. Tarim and Smith (2008) give a computationally efficient constraint programming formulation. An exact formulation and a solution method are presented in Rossi et al. (2008).

All the above mentioned works assume either zero or a fixed (deterministic) supplier lead-time (i.e., replenishment lead-time). However, the lead-time uncertainty, which in various industries is an inherent part of the business environment, has a detrimental effect on inventory systems. For this reason, there is a vast inventory control literature analysing the impact of supplier lead-time uncertainty on the ordering policy (Whybark and Williams, 1976; Speh and Wagenheim, 1978; Nevison and Burstein, 1984). A comprehensive discussion on stochastic supplier lead-time in continuous-time inventory systems is presented in Zipkin (1986). Kaplan (1970) characterises the optimal policy for a dynamic inventory problem where the lead-time is a discrete random variable with known distribution and the demands in successive periods are assumed to form a stationary stochastic process. Since tracking all the outstanding orders through the use of dynamic programming requires a large multi-dimensional state vector, Kaplan assumes that orders do not

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cross in time and supplier lead time probabilities are independent of the size/number of outstanding orders (for details on order-crossover see Hayya et al., 1995).

The assumption that orders do not cross in time is valid for systems where the supplier production system has a single-server queue structure operating under a FIFO policy. Nevertheless, there are settings in which this assumption is not valid and orders cross in time. This has been recently investigated in Hayya et al. (2008), Bashyam and Fu (1998) and Riezebos (2006). As Riezebos underscores, the types of industries that have a higher probability of facing order crossovers are either located upstream in the supply chain, or use natural resources, or order strategic materials from multiple suppliers or from abroad. In a case study, he showed that the potential cost savings realized by taking order crossovers into account were approximately 30%. Unfortunately, he remarks, modern ERP systems are not able to handle order crossovers effectively.

In a recent work, Babaï et al. (2009) analyze a dynamic re-order point control policy for a single-stage, single-item inventory system with non-stationary demand and lead-time uncertainty. To the best of our knowledge, there is no complete or heuristic approach in the literature that addresses the computation of (R^n,S^n) policy parameters under stochastic supplier lead time and service level constraints. Computing optimal policy parameters under these assumptions is a hard problem from a computational point of view. We argue that incorporating both a non-stationary stochastic demand and a stochastic supplier lead time—without assuming that orders do not cross in time—in an optimization model is a relevant and novel contribution.

In this work, we propose a stochastic constraint programming (Walsh, 2002) model for computing optimal (R^n,S^n) policy parameters under service level constraints and stochastic supplier lead times. In stochastic constraint programming, complex non-linear relations among decision and stochastic variables—such as the chance-constraints that enforce the required service level—can be effectively modeled by means of *global chance-constraints* (Hnich et al., 2009). Examples of global chance-constraints applied to inventory control problems can be found in Rossi et al. (2008) and Tarim et al. (2009). Our model incorporates a dedicated global chance-constraint that enforces, for each replenishment cycle scheduled, the required non-stock-out probability. The model is tested on a set of instances that are solved to optimality under a discrete stochastic supplier lead time with known distribution.

The paper is organized as follows. In Section 2 we provide the formal definition of the problem and we discuss the working assumptions. In Section 3 we provide a deterministic reformulation for the chance-constraints that enforce the required service level. In Section 4 we introduce stochastic constraint programming and we discuss how it is possible to embed the deterministic reformulation of the chance-constraints within a global chance-constraint. This global chance-constraint is then enforced in the stochastic constraint programming model for computing the optimal policy parameters. In Section 5 we present our computational experience on a set of instances. Finally, in Section 6, we draw conclusions.

2. Problem definition

We consider the uncapacitated, single location, single product inventory problem with a finite planning horizon of N periods and a demand d_t for each period $t \in \{1,\ldots,N\}$, which is a random variable with probability density function $g_t\,(d_t)$. We assume that the demand occurs instantaneously at the beginning of each time period. The demand we consider is non-stationary, that is it can vary from period to period, and we also assume that demands in different periods are independent.

Following Eppen and Martin (1988), an order placed in period t will be received after l_t periods, where l_t is a discrete random variable with probability mass function $f_t(\cdot)$. This means that an order placed in period t will be received after k periods with probability $f_t(k)$. We shall assume that there is a maximum lead-time L for which $\sum_{k=0}^{L} f_t(k) = 1$. Therefore the possible lead-time lengths are limited to $\Lambda = \{0,\ldots,L\}$ and the probability mass function is defined on the finite set Λ. Note that lead-times are mutually independent and each of them is also independent of the respective order quantity.

A fixed delivery cost a is incurred for each order. A linear holding cost h is incurred for each unit of product carried in stock from one period to the next. Without loss of generality, we will adopt the following assumption that concerns the accounting of inventory holding costs: we will charge an inventory holding cost at the end of each period based on the current inventory position, rather than the current inventory level. This will reflect the fact that interests are charged not only on the actual amount of items in stock, but also on outstanding orders. Doing so often makes sense since companies may assess holding cost on their total invested capital and not simply on items in stock. A further and detailed justification for this can be found in Hunt (1965).

We assume that it is not possible to sell back excess items to the vendor at the end of a period and that negative orders are not allowed, so that if the actual stock exceeds the order-up-to-level for that review, this excess stock is carried forward and not returned to the supply source. However, such occurrences are regarded as rare events (see the discussion in Bookbinder and Tan, 1988 and Tarim and Kingsman, 2004) and accordingly the cost of carrying this excess stock and its effect on the service levels of subsequent periods are ignored.

As a service level constraint we require that, with a probability of at least a given value α, at the end of each period the net inventory will be non-negative. Our aim is to minimize the expected total cost, which is composed of ordering costs and holding costs, over the N-period planning horizon, satisfying the service level constraints by fixing the future replenishment periods and the corresponding order-up-to-levels at the beginning of the planning horizon.

The actual sequence of actions is adopted from Kaplan (1970). At the beginning of a period, the inventory on hand after all the demands from previous periods have been realized is known. Since we are assuming complete backlogging, this quantity may be negative. Also known are orders placed in previous periods which have not been delivered yet. On the basis of this information, an ordering decision is made for the current period. All the deliveries that are to be made during a period are assumed to be made immediately after this ordering decision and hence are on hand at the beginning of the period. In summary, there are three successive events at the beginning of each period. First, stock on hand and outstanding orders are determined. Second, an ordering decision is made on the basis of this information. Third, all supplier deliveries for the current period, possibly including the most recent orders, are received.

3. Non-stationary stochastic lead-time

Let us denote the inventory position (the total amount of stock on hand plus outstanding orders minus back-orders) at the end of