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A generalized algebraic model for optimizing inventory decisions in a centralized or decentralized multi-stage multi-firm supply chain

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ABSTRACT

First of all, a number of integrated models with/without lot streaming under the integer multiplier coordination mechanism is generalized by allowing lot streaming and three types of inspection for some/all upstream firms. Secondly, the optimal solutions to the three- and four-stage models are individually derived, both using the perfect squares method, which is a simple algebraic approach so that ordinary readers unfamiliar with differential calculus can easily understand how to obtain the optimal solution procedures. Thirdly, optimal expressions for some well-known models are deduced. Fourthly, expressions for sharing the coordination benefits based on Goyal's (1976) scheme are derived, and a further sharing scheme is introduced. Fifthly, two numerical examples for illustrative purposes are presented. Finally, some future research works involving extension or modification of the generalized model are suggested.

1. Introduction

Supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of material, information and capital between suppliers of raw materials and the ultimate customers. The benefits of a property managed supply chain include reduced costs, faster product delivery, greater efficiency, and lower costs for both the business and its customers. These competitive advantages are achieved through improved supply chain relationships and tightened links between chain partners such as suppliers, manufacturing facilities, distribution centers, wholesalers, and end users (Berger et al., 2004). Besides integrating all members in a supply chain, to improve the traditional method of solving inventory problems is also necessary. Without using derivatives, Grubbström (1995) first derived the optimal expressions for the classical economic order quantity (EOQ) model using the unity decomposition method, which is an algebraic approach. Adopting this method, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) respectively derived the optimal expressions for an EOQ and economic production quantity (EPQ) model with complete backorders. In this paper, a generalized model for a three- or four-stage multi-firm production-inventory integrated system is solved using the perfect squares method adopted in Leung (2008a,b, 2009a,b, 2010), which is also an algebraic approach; whereby optimal expressions of decision variables and the objective function are derived.

Assume that there is an uninterrupted production run. In the case of lot streaming in stage $i(=1,\ldots,n-1)$, shipments can be made from a production batch even before the whole batch is finished. According to Joglekar (1988, pp. 1397–8) the average inventory with lot streaming, for example, in stage 2 of a three-stage supply chain, is $\frac{T_3D_{2j}}{2}[\varphi_{2j}+(K_2-1)\bar{\varphi}_{2j}]$ units, which is the same as Eq. (7) of Ben-Daya and Al-Nassar (2008).

However, some or all suppliers/manufacturers/assemblers cannot accommodate lot streaming because of regulations, material handling equipment, or production restrictions (Silver et al., 1998, p. 657). Without lot streaming, no shipments

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can be made from a production batch until the whole batch is finished. Sucky (2005) discussed the integrated single-vendor single-buyer system, with and without lot streaming, in detail. The opportunity of lot streaming affects supplier's average inventory. According to Goysl's (1988, p. 237), the average inventory without lot streaming, for example, in stage 2 of a 3-stage supply chain, is $\frac{T_3 D_{2j}}{2}(\varphi_{2j}K_2 + K_2 - 1)$ units, which is the same as term 2 in Eq. (5) of Khouja (2003).

In the inventory/production literature, all researchers have constructed their models under the assumption of either allowing lot streaming for all firms involving production (Khouja, 2003) or not (Ben-Daya and Al-Nassar, 2008), or both extremes (Sucky, 2005 and Leung, 2010). The main purpose of the paper is twofold: First, we build a generalized model incorporating a mixture of the two extremes, and solve it algebraically. As a result, we can deduce and solve such special models as Khouja (2003), Cárdenas-Barrón (2007), Ben-Daya and Al-Nassar's (2008), Seliaman and Ahmad's (2009), and Leung (2009a). In addition, with appropriate assignments as in Section 5 of Leung (2010), we can also deduce and solve other special models: Yang and Wee (2002), Wu and Ouyang (2003) or Wee and Chung (2007), and Chung and Wee (2007). Second, we derive expressions for sharing the coordination benefits based on Goyal's (1976) scheme, and introduce a further sharing scheme.

Some good review articles exist that provide an extensive overview of the topic under study and can be helpful as guidance through the literature. We mention surveys by Goyal and Gupta (1989), Goyal and Deshmukh (1992), Bhatnagar et al. (1993), Maloni and Benton (1997), Sarmah et al. (2006), and Ben-Daya et al. (2008). The well-known models of Goyal (1976), Banerjee (1986), Lu (1995), and Hill (1997) are extended by Ben-Daya et al. (2008) as well. Other recently related articles include Chan and Kingsman (2007), Chiou et al. (2007), Cha et al. (2008), Leng and Parlar (2009a,b), and Leng and Zhu (2009).

2. Assumptions, symbols and designations

The integrated production-inventory model is developed under the following assumptions:

(1) A single item is considered.
(2) There are two or more stages.
(3) Production and demand rates (with the former greater than the latter) are independent of production or order quantity, and are constant.
(4) Unit cost is independent of quantity purchased, and an order quantity will not vary from one cycle to another.
(5) Neither a wait-in-process unit, nor a defective-in-transit unit, is considered.
(6) Each upstream firm implements perfect inspection to guarantee that defective units are not delivered to any retailer. Three types of inspection suggested in Wee and Chung (2007) are executed.
(7) Each type of inspection costs is different for all firms in each stage involving production.
(8) Setup or ordering costs are different for all firms in the chain.
(9) Holding costs of raw materials are different from those of finished products.
(10) Holding costs of raw materials are different for all firms in the chain.
(11) Holding costs of finished goods are different for all firms in the chain.
(12) Lot streaming is allowed for some firms but no lot streaming is allowed for the rest in each stage involving production.
(13) No shortages are allowed for any retailer.
(14) All firms have complete information of each other.
(15) The number of shipments of each supplier, manufacturer, assembler or retailer is a positive integer.
(16) The planning horizon is infinite.

The following symbols (some as defined in Leung, 2009a) are used in the expression of the joint total relevant cost per year.

D_{ij} = demand rate of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [units per year]
P_{ij} = production rate of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [units per year]
g_{ij} = holding cost of incoming raw material of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [$ per unit per year]
h_{ij} = holding cost of finished goods of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [$ per unit per year]
S_{ij} = setup or ordering cost of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [$ per cycle]
A_{ij} = inspection cost per cycle of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [$ per cycle]
B_{ij} = inspection cost per delivery of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [$ per delivery]
C_{ij} = inspection cost per unit of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [$ per unit]

For a centralized supply chain (or the integrated approach), we have

$T_{nj} = T_n$ = basic cycle time of firm $j(=1,\ldots,J_n)$ in stage n (T is a decision variable with *non-negative* real values) [a fraction of a year]
$T_{ij} = T_n\prod_{k=i}^{n-1}K_k = T_n\prod_{k=i}^{n}K_k$ with $K_n \equiv 1$ = integer multiplier cycle time of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ ($K_1,\ldots,K_{n-1}$ are decision variables, each with *positive* integral values) [a fraction of a year]