Int. J. Production Economics 128 (2010) 351–357

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Buyer–vendor inventory coordination with quantity discount incentive for fixed lifetime product

Yongrui Duan [a], Jianwen Luo [b,*], Jiazhen Huo [a]

[a] School of Economics and Management, Tongji University, Shanghai 200092, PR China
[b] School of Management, Shanghai Jiao Tong University, Shanghai 200052, PR China

ARTICLE INFO

Article history:
Received 26 August 2008
Accepted 8 July 2010
Available online 23 July 2010

Keywords:
Quantity discount
Fixed lifetime product
Coordination
Inventory
Supply chain

ABSTRACT

In this paper, a single-vendor, single-buyer supply chain for fixed lifetime product is considered. We propose models to analyze the benefit of coordinating supply chain through quantity discount strategy. Under the proposed strategy, the buyer is requested to alter his current order size such that the vendor can benefit from lower costs, and quantity discount is offered by the vendor to compensate the buyer for his increased inventory cost, and possibly provide an additional savings. In addition, the centralized decision-making model is formulated to examine the effectiveness of the proposed quantity discount model. It is proved that the quantity discount strategy can achieve system optimization and win–win outcome. At last, a detailed numerical example is presented to illustrate the benefit of the proposed strategy.

1. Introduction

Perishability of either raw materials or finished products is a major problem in some industries such as agro-food industry, drug industry. Due to the limited product lifetime, an ineffective inventory management at each stage in the supply chain from production to consumers can lead to high system costs including ordering costs, shortage costs, inventory holding costs, and outdating costs. Moreover, the quality of products (freshness) may be unacceptable, thus reducing customer satisfaction. Liu and Shi (1999) classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Decay models deal with inventory that shrinks continuously and proportionally with time, while finite lifetime models assume a limited lifetime for each item. Furthermore, the finite lifetime models can be generally classified into two subcategories, namely common or fixed finite lifetime models and random finite lifetime models. Items with common finite lifetimes, usually referred in the literature as perishable items (Liu and Lian, 1999), perish at the same age if not used by demand. Fresh products, cans of fruit, foodstuffs, and drugs are examples of the items having fixed finite lifetimes. The random finite lifetimes, on the other hand, are treated as random variables with certain probability distributions, such as exponential and Erlang. Items with random lifetimes, thus, spoil at different ages.

Past researches on the Fixed-Life Perishable Problem (FLPP), such as Fries (1975), Nandakumar and Morton (1993), Liu and Lian (1999), and Lian and Liu (2001), mainly addressed single-stage inventory systems. Fujiwara et al. (1997) studied the problem of ordering and issuing policies in controlling finite-lifetime fresh-meat-carcass inventories in the supermarket. Kanchana and Anulark (2006) investigated the effect of product perishability and retailers' stockout policy on system total cost, net profit, service level and average inventory level in a two-echelon inventory system, and a periodic review inventory-distribution model was proposed to deal with the case of fixed-life perishable product.

Because members of a supply chain are different entities with their own interests, active cooperation and close coordination play an important role in supply chain management. Therefore, some efficient mechanisms are necessary to enforce coordination between parties in the supply chain. Examples of such mechanisms include quantity discount (Goyal and Gupta, 1989), revenue sharing (Giannoccaro and Pontrandolfo, 2004), sales rebate (Wong et al., 2009), trade credit (Chen and Kang, 2010). Among these mechanisms, quantity discount is a commonly used scheme. Goyal and Gupta (1989) reviewed the literatures on the quantity discount models. For fixed lifetime items, there are few literatures on the coordination mechanisms.

In this paper, a single-vendor, single-buyer supply chain for item with fixed lifetime is considered. We develop models to analyze the benefit of coordinating supply chain by quantity discount strategy. If coordination is not introduced, given buyer's EOQ order quantity, the vendor's order size is an integer multiple

* Corresponding author. Tel.: +86 21 52302053.
E-mail address: jwluo@sjtu.edu.cn (J. Luo).

0925-5273/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2010.07.030

of the buyer's that minimizes his own inventory cost. Under the proposed coordination strategy, the vendor requests the buyer to alter his current EOQ, and the vendor's order size is another integer multiple of the buyer's new order quantity such that the vendor can benefit from lower setup ordering and inventory holding costs. To entice the buyer to accept this strategy, the vendor must compensate the buyer for his increased inventory cost, and possibly provide an additional saving by offering the buyer a quantity discount, which depends on his order size.

The rest of this paper is organized as follows. In Section 2, the decentralized models with and without coordination, and centralized model are formulated. The analytically tractable solutions to these models are obtained. It is proved that the quantity discount strategy can achieve system optimization and win–win outcome. A numerical example is presented in Section 3 to illustrate the effectiveness of the proposed quantity discount strategy. The summary and concluding remark are presented in the last section.

2. Model formulation

In this section, models without coordination and with quantity discount coordination in decentralized decision-making scenario and system optimization model in centralized decision-making scenario are formulated. Some assumptions for our models are: (i) shortage is not allowed; (ii) lead time is zero; (iii) all items ordered by the vendor arrive new and fresh, that is, their age equals zero.

The notations used in this paper are as follows:

D	annual demand of the buyer;
L	lifetime of product;
A_1 and A_2	denote the vendor and buyer's setup costs per order, respectively;
h_1 and h_2	denote the vendor and buyer's holding costs, respectively;
p_1 and p_2	denote the delivered unite price paid by the vendor and the buyer, respectively;
Q_0	buyer's *EOQ*;
m	denotes the vendor's order multiple in the absence of any coordination;
K	denotes the buyer's order multiple under coordination, KQ_0 buyer's new order quantity;
$d(K)$	denotes the per unit dollar discount to the buyer if he orders KQ_0 every time;
n	denotes the vendor's order multiple under coordination.

2.1. Model formulation for system without coordination

In the absence of any coordination, the buyer's order quantity is simply the *EOQ*, i.e. $Q_0=\sqrt{2DA_2/h_2}$, with the annual cost $TC_r=\sqrt{2DA_2h_2}$. The vendor's order size should be some integer multiple of Q_0 denoted by mQ_0, since he is faced with a stream of demands at fixed intervals $t_0=Q_0/D=\sqrt{2A_2/Dh_2}$.

In this case, the vendor's average inventory held up per year is

$$[(m-1)Q_0+(m-2)Q_0+\cdots+Q_0+0Q_0]/m=(m-1)Q_0/2.$$

The total annual cost for the vendor is given by

$$TC_v(m)=\frac{DA_1}{mQ_0}+\frac{(m-1)Q_0h_1}{2}$$

$$=\frac{DA_1}{m\sqrt{\frac{2DA_2}{h_2}}}+\frac{(m-1)\sqrt{\frac{2DA_2}{h_2}}h_1}{2}$$

$$=\frac{A_1}{m}\sqrt{\frac{Dh_2}{2A_2}}+(m-1)h_1\sqrt{\frac{DA_2}{2h_2}}.$$

So the vendor's problem without coordination can be formulated as follows:

$$\min\quad TC_v(m)$$
$$\text{s.t.}\quad \begin{cases} mt_0\le L,\\ m\ge 1,\end{cases}\tag{1}$$

where $mt_0\le L$ is to ensure that items are not overdue before they are used up (sold up) by the buyer.

Theorem 1. *Let m^* be the optimum of* (1), *if $L^2\ge 2A_2/Dh_2$, then*

$$m^*=\min\left\{\left\lceil\sqrt{\frac{A_1h_2}{A_2h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil,\left\lfloor\frac{L}{\sqrt{\frac{2A_2}{Dh_2}}}\right\rfloor\right\},\tag{2}$$

where $\lceil x\rceil$ is the least integer greater than or equal to x, $L^2\ge 2A_2/Dh_2$ is to ensure that $m^*\ge 1$.

Proof. Since

$$\frac{d^2TC_v(m)}{dm^2}=\frac{2A_1}{m^3}\sqrt{\frac{Dh_2}{2A_2}}>0,$$

$TC_v(m)$ is strictly convex in m. Let m_1^* be the optimum of $\min_{m\ge1}TC_v(m)$, then

$$m_1^*=\max\{\min\{m|TC_v(m)\le TC_v(m+1)\},1\}$$
$$=\max\left\{\min\left\{m|m(m+1)\ge\frac{2DA_1}{Q_0^2h_1}\right\},1\right\}$$
$$=\left\lceil\sqrt{\frac{A_1h_2}{A_2h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil\ge 1.$$

By substituting $t_0=\sqrt{2A_2/Dh_2}$ into the constraint in (1), the following inequality holds:

$$m\sqrt{\frac{2A_2}{Dh_2}}\le L.$$

Set

$$m_2^*=\left\lfloor\frac{L}{\sqrt{\frac{2A_2}{Dh_2}}}\right\rfloor.$$

Since $L^2\ge 2A_2/Dh_2$, $m_2^*\ge 1$ holds. In view of $TC_v(m)$ is a convex function, if $m_1^*\le m_2^*$, $m^*=m_1^*$, else $m^*=m_2^*$. So if $L^2\ge 2A_2/Dh_2$, $m^*=\min\{m_1^*>,m_2^*\}$. The proof of Theorem 1 is complete. □

Remark 1. In the absence of any coordination, the vendor's order size is $m^*\sqrt{2DA_2/h_2}$, and place $D/m^*\sqrt{2DA_2/h_2}$ orders each year with an interval of $m^*\sqrt{2DA_2/h_2}/D$ throughout that time. The minimized total cost is $TC_v(m^*)$.

2.2. Model formulation for system with coordination

Under the quantity discount coordination strategy, the vendor requests the buyer to alter his current order size by a factor $K(K>0)$, and compensate the buyer by a quantity discount at a