



# A single-item continuous review inventory problem with space restriction

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## ABSTRACT

With today's high cost of land acquisition observed in most countries, management can no longer afford large storage facilities to house their products. For this reason, one of the main concerns of inventory managers is to ensure that enough storage space is available upon the delivery of a product. They, therefore, have to determine the least cost ordering policy so as to meet demand and taking into account the space limitation. In this paper, we investigate the problem of a single item continuous review inventory problem in the presence of a space restriction when the demand is stochastic. Through a sensitivity analysis study, it is found that ordering cost, storage capacity, average demand per unit time, and holding cost rate are the most influential problem parameters on the expected inventory costs. It is also shown numerically that a simple solution based on economic order quantity is performing very well with an average and maximum cost penalty of 0.19% and 2.03%, respectively.

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## 1. Introduction

The problem addressed in this paper is of critical importance to the efficiency of warehousing operations. In fact, it is often the case that upon the receipt of a product, the quantity received is larger than the space available in the storage location assigned to the product. This situation is more serious when a dedicated storage method is used to assign products to storage areas upon their delivery. In case of insufficient storage space the over-ordered quantity is either returned to the supplier or stored in a non-dedicated area with different storage conditions and, therefore, could be subject to spoilage. Either option will result in additional costs caused by the penalty cost charged by the supplier or the costs of extra order picking and material handling operations.

Inventory decision makers are often confronted with situations, such as the one discussed above, where they have to determine ordering policies that make the best use of limited resources. Storage space is one of the scarcest resources that affect the efficiency of inventory control policies. Consequently, management's concern is to ensure that there is enough space to accommodate the product upon its receipt. Therefore, the ordering quantity of the product is limited by the free space in the storage facility at the time of delivery. This amount of space is not known to the decision maker when placing an order since the demand over the lead time is, in most practical cases, random. The objective of this paper is to address the above management

concern by proposing a continuous review ( $Q, R$ ) ordering policy that takes into account the space limitation.

The continuous review ( $Q, R$ ) single product, single location model with stochastic stationary demand and infinite planning horizon and its variants have received considerable attention from researchers in the area of inventory control (Gerchak and Parlar, 1991; Lee and Nahmias, 1993; Johansen and Thorstenson, 1998; Cakanyildirim et al., 2000; Lee and Schwarz, 2007; Thiel et al., 2010). Under this type of review policy, an order of size  $Q$  is placed whenever the inventory position (on-hand – backorder level + on order quantity) drops to the reorder point,  $R$ . The ordering quantity is delivered after a period of time, called lead time, has elapsed. Therefore, the reorder point quantity should be enough to satisfy the random demand during lead time, otherwise a stockout situation will be observed. The optimal ( $Q, R$ ) ordering policy is the one that minimizes the expected total cost composed of the expected ordering, holding and shortage costs. Accurate (Zheng, 1992) as well as approximate (see e.g., Hadley and Whitin, 1963; Wagner, 1975; Love, 1979) expressions of the expected holding cost have been proposed in the stochastic inventory literature. However, given that the space constraint is imposed on the on-hand inventory (a random variable) and not on the inventory position (which is uniformly distributed between  $R$  and minimum of  $R+Q$  and available space), a derivation of an exact expression for the inventory holding cost can be very difficult to develop. Therefore, we follow Johnson and Montgomery (1974) heuristic treatment in deriving the mathematical expression of the expected total cost per cycle.

Compared to the literature of space constrained inventory models with deterministic demand, very few papers have been published for stochastic demand models. In fact, previous

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research on inventory problems with space restriction focused on the case of multiple items with deterministic demands. Hariga and Jackson (1995) provide comprehensive review of this literature up to 1995. Haksever and Moussourakis (2005) review also the literature of deterministic multi-product multi-constraint inventory systems with stationary ordering policies. Veinott (1965) is among the first authors to address the stochastic version of inventory problems with storage space limitation. He establishes conditions for the base-stock ordering policy to be optimal in a multi-item periodic review system with a finite horizon and zero ordering cost. Ignall and Veinott (1969) show that the myopic ordering policy is optimal when the demand is stationary. Using a dynamic programming approach, Beyer et al. (2001) obtain the optimal ordering policy for the stochastic multi-product problem with finite and infinite horizon as well as stationary and non-stationary discounted costs. Jeddi et al. (2004) study a multi-item continuous review system with random demand subject to a budget constraint when the payment is due upon order arrival. Our model formulation differs from the one developed in Jeddi et al.'s paper since the approximate mathematical expression of the inventory holding cost (Hadley and Whitin, 1963) is affected by the storage space limitation, which was not the case in the presence of a budget constraint. Minner and Silver (2005) study the space/budget constrained multi-product continuous review problem with zero lead times and non-allowed backorders. They show that under Poisson demand the problem can be formulated as a semi-Markov decision process and can be solved optimally for small sized problems. Recently, Xu and Leung (2009) propose an analytical model in a two-party vendor managed system where the retailer restricts the maximum space allocated to the vendor. They adopt the order-up-to-S stocking policy to generate the optimal inventory decision under this space restriction.

In this paper, we address space constrained continuous review problem for a single item with random demand and positive lead time. The remainder of the paper is organized as follows. In the next section, we present the assumptions and notation used to develop the mathematical model. In the third section, we develop the mathematical model. In the fourth section, we illustrate the model with the special case of normally distributed lead time demand. In the same section, we conduct a sensitivity analysis to study the effects of the problem parameters on the cost output. Finally, the last section concludes the paper.

## 2. Model assumptions and notation

The mathematical model developed in this paper is based on the following assumptions:

1. Inventory position is continuously monitored and an order of size  $Q$  is made whenever the inventory level hits the reorder point,  $R$ .
2. Units are demanded in small quantities, so that overshooting of the reorder point is not appreciable.
3. There are no orders outstanding at the time the reorder point is reached.
4. The lead time demand,  $x$ , follows a stationary probability distribution.
5. The backorder cost is time invariant.
6. An area with limited space,  $W$ , is reserved for the storage of the product.
7. The over-ordered quantity that cannot be accommodated in the available space at the delivery time is returned to the supplier
8. The supplier charges  $100\alpha\%$  of the unit purchasing cost for each unit of the product returned because of over-ordering.

9. The reorder level is larger than the mean of the lead time demand.
10. The ordering quantity is smaller than the storage space capacity.
11. The time the system is out of stock during a cycle is small compared to the cycle length.

Note that the first five assumptions are the ones that are made for the classical continuous review ( $Q, R$ ) system. According to the seventh assumption, the only available option to face an over-ordering situation is to return the over-ordered quantity to the supplier. The ninth assumption is made to ensure that safety stock is positive. The tenth assumption is made as one would not order a quantity beyond the space capacity knowing that there is a positive probability that the storage area will not be completely free at the delivery time. Finally, based on the last assumption, the realized lead time demands in different inventory cycles are approximately equal.

We will also use the following additional notation throughout the paper:

$x$	lead time demand, a random variable
$Y$	the actual quantity delivered given a realized demand, $x$
$C$	unit cost of the item
$h$	inventory holding cost per unit per unit time
$A$	fixed ordering cost per order
$\pi$	backorder cost per unit short
$D$	expected demand per unit time
$\mu$	expected demand during the lead time
$\sigma$	standard deviation of the demand during the lead time
$F(x)$	cumulative distribution function of the lead time demand
$B_{RQ}$	$Q+R-W$
$S_R$	$\int_R^\infty (x-R)dF(x)$ expected number of units short per cycle

Additional notation will be introduced later when needed.

## 3. Model formulation

When an order quantity of size  $Q$  is placed, the actual quantity unloaded into the storage facility depends on the inventory level (a random variable) immediately after the receipt of the order. In fact, three different cases have to be distinguished depending on the values of lead time demand, the reorder point, the inventory level just after the receipt of an order and the storage capacity,  $W$ . In the following, we will develop expression for the cycle time and the inventory cost per cycle for each of these three cases. As in Hadley and Whitin (1963), a cycle is defined as the time interval between the receipts of two consecutive orders. Therefore, the order receipt times constitute the regeneration points of the renewal process.

Case 1:  $R-x \geq 0$  and  $R-x+Q \leq W$ .

The first condition states that the quantity demanded during the lead time is smaller than the reorder point. In other words, the minimum inventory level during the cycle, which occurs immediately before receipt of an order, is nonnegative and equal to  $R-x$ . According to the second condition, the maximum inventory level during the cycle,  $R-x+Q$ , which takes place immediately after the order arrives, is smaller than the storage capacity. Therefore, for a realized lead time demand,  $x$ , we average the minimum and maximum inventory levels to approximate the area under the inventory level curve over one cycle, which is expressed as

$$Area_1 = \frac{Q}{D} \left( \frac{Q}{2} + R - x \right)$$

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