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# Coordinated scheduling of production and delivery stages with stage-dependent inventory holding costs

Ik Sun Lee [a], S.H. Yoon [b,*]

[a] School of Business, Dong-A University, Seo-gu, Busan 602-760, Republic of Korea
[b] School of Management, Yeungnam University, Republic of Korea

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ABSTRACT

This paper considers a problem of integrated decision-making for job scheduling and delivery batching wherein different inventory holding costs between production and delivery stages are allowed. In the problem, jobs are processed on a facility at a production stage and then delivered at the subsequent delivery stage by a capacitated vehicle. The objective is to find the coordinated schedule of production and delivery that minimizes the total cost of the associated WIP inventory, finished product inventory and delivery, where both the inventory costs are characterized in terms of the weighted flow-time and the delivery cost is proportional to the required number of delivery batches. It is proved that the problem is NP-hard in the strong sense. Thereupon, three heuristic algorithms are derived. Some restricted cases are also characterized as being solvable in polynomial time. Numerical experiments are conducted to evaluate the performance of the derived heuristic algorithms.



## 1. Introduction

This paper considers a problem of integrated decision-making for job scheduling and delivery batching wherein different (stage-dependent) inventory holding costs between production and delivery stages are allowed. In the problem, jobs are processed on a production facility and then delivered to a customer by a capacitated vehicle. The customer can be interpreted as a distribution center (or, warehouse). Therefore, all the completed jobs from the production facility are frequently delivered in batches to the distribution center. The objective is to find the coordinated schedule of production and delivery that minimizes the total cost of the associated inventory and delivery.

The inventory cost is generally computed by the multiplication of the unit holding cost and the flow-time spent in the system. The holding cost represents a combination of the cost of capital, the cost of physical storage and the cost of losses due to spoilage; hence, it highly depends on the inventory type (or value). Therefore, this paper explicitly distinguishes between two types of inventory cost. The first is incurred by WIP (work-in-process) and the second is incurred by finished product. To compute the inventory costs, this paper incorporates two types of flow-time: production flow-time and delivery flow-time. The production flow-time of each job represents the time until the processing of the job is completed. The WIP-inventory cost is assumed to be proportional to the production flow-time. The finished jobs are stored and then delivered to the customer. Therefore, the delivery flow-time of each job can be measured by the time that lapses from the completion of production to the completion of delivery. It is noted that delivery is commonly made in batches subject to a given vehicle capacity so that the delivery flow-time of each job depends on the associated delivery schedule. The finished-product inventory cost is incurred during the delivery flow-time. Thus, the overall inventory cost is measured by the weighted sum of the two flow-times, where the weights are represented by their respective unit holding costs. Moreover, the delivery cost is modeled to be proportional to the number of deliveries that are to be made for all the jobs.

In the literature, much research has focused on the area of integrating job scheduling and delivery batching together, under various assumptions and objective measures that differ from the problem proposed in this paper. For example, Potts [1] and Hall and Shmoys [2] have studied scheduling problems with non-identical job-release times and delivery times, under the assumption that a sufficient number of vehicles are available to deliver the jobs. Lee and Chen [3], Sung and Kim [4,5], Chang and Lee [6] and Li et al. [7] considered capacity restrictions on delivery batches, but tried to only minimize some scheduling objectives, without considering any delivery costs, including the makespan, the sum of completion times, the maximum lateness, the number of tardy jobs and the total tardiness. Pundoor and Chen [8] considered the maximum tardiness and delivery cost together.

* Corresponding author. Tel.: +82 53 810 2728.
*E-mail addresses:* lis1007@dau.ac.kr (I. Sun Lee), shyoon@ynu.ac.kr (S.H. Yoon).

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Lee and Chen [3], Sung and Kim [4,5], Pundoor and Chen [8] and Li et al. [7] considered the situation of identical job-sizes, but Chang and Lee [6] considered the case where job-sizes varied.

Yang [9] and Hall et al. [10] analyzed various production-and-delivery scheduling problems with fixed batch-delivery time-points, under the assumptions of infinite vehicle capacities, a sufficient number of vehicles and no delivery costs. Herrmann and Lee [11], Yuan [12], Chen [13], Cheng et al. [14], Hall and Potts [15], Lin et al. [16], Yan and Tang [17] and Day et al. [18] analyzed various machine-scheduling problems for jobs to be delivered in batches following job-processing, under the assumption of infinite vehicle capacities but with a charge for delivery. Qi [19] considered a situation where the raw material for processing jobs is delivered in batches to a single machine and the raw material delivery and job sequencing decisions are made simultaneously. He derived a branch-and-bound algorithm to minimize the sum of the delivery and flow-time costs. Chen [20] has published a recent review of the research on production-and-delivery scheduling problems.

All the above cited references assumed that the inventory holding costs per unit-time at the production and delivery stages are the same. However, the inventory holding cost per unit-time of each finished product is greater than that of any intermediate product, which suggests that it may be appropriate to model the inventory holding cost as being stage-dependent. Stage-dependent inventory holding costs are expected to make the coordination between production and delivery more effective.

The organization of this paper is as follows. Section 2 describes the problem. Sections 3 and 4 characterize some solution properties, upon which three heuristic algorithms are derived. Some numerical experiments are conducted to evaluate the performance of the derived heuristic algorithms. Some restricted cases are also characterized as being solvable in polynomial time. Section 5 makes some concluding remarks.

## 2. Problem description

The proposed problem is now described in detail. There are $n$ jobs (products) to be processed on a single facility. Job $j$ (numbered $j=1,2,\ldots,n$) requires a processing time $p_j$. All the jobs are assumed to be continuously processed so that no idle time is inserted between jobs. This assumption makes sense in many production situations where the setup and opportunity costs incurred by forced idleness are high. The setup cost is related to activities, such as heating, cooling and replacing, which are associated with the restarting of the facility after idleness and the opportunity cost represents the loss of utilization of the facility and labor. After the processing of a job is completed, the job is stored at a storage center and delivered to the customer. A single vehicle of capacity $c$ delivers all the jobs. It is assumed that all jobs have the same size, so that the vehicle capacity represents the number of jobs that can be loaded in each shipment. We respectively, denote by $n_b$ and $\delta$ the number of delivery batches and the fixed cost (charge) that is incurred for the delivery of each batch. A delivery-batch is defined as a group of jobs that are to be delivered together in one shipment. The vehicle transshipment time from the storage center to the customer is a constant, $d_1$ and the time for the vehicle to return from the customer to the storage center is a constant $d_2$, where $d_1$ and $d_2$ are independent of the delivery quantity. It is assumed that $p_j$, $n_b$, $d_1$, $d_2$ and $c$ are integral. Let $F_j^w$ and $F_j^f$, respectively, denote the production flow-time and the delivery flow-time of job $j$. Each job's delivery flow-time is composed of its waiting time at the storage center and its transshipment time, $d_1$. Let $h_w$ and $h_f$, respectively, denote the WIP holding cost per unit production flow-time and the finished-product holding cost per unit delivery flow-time. The proposed problem is depicted in Fig. 1.

The standard classification scheme for scheduling problems (Pinedo [21]) $\alpha|\beta|\gamma$ is adapted, where $\alpha$ indicates the machine environment, $\beta$ describes certain characteristics or restrictive requirements and $\gamma$ defines the objective function to be minimized. Since this paper considers a single facility, the first field, $\alpha$, is fixed to 1. For $\beta$, this paper considers two different cases, $h_w < h_f$ and $h_w \geq h_f$. The first case ($h_w < h_f$) seems to be more practical, since the cost per unit-time of finished-product inventory is generally greater than that of WIP inventory. For the sake of completeness, the other case ($h_w \geq h_f$) is also considered. Finally, for $\gamma$, the objective function "$OC$" of the proposed problem is represented as follows.

$$OC = h_w \sum_{i=1}^{n} F_i^w + h_f \sum_{i=1}^{n} F_i^f + \delta \cdot n_b, \tag{1}$$

## 3. Case $h_w \geq h_f$

To find the optimal coordinated schedule for the problem $1|h_w \geq h_f|OC$, some solution properties are characterized in Theorems 1 and 2.

**Theorem 1.** *For the problem $1|h_w \geq h_f|OC$, at the production stage, it is optimal to process the jobs in the SPT (shortest processing time) order, regardless of their delivery schedules.*

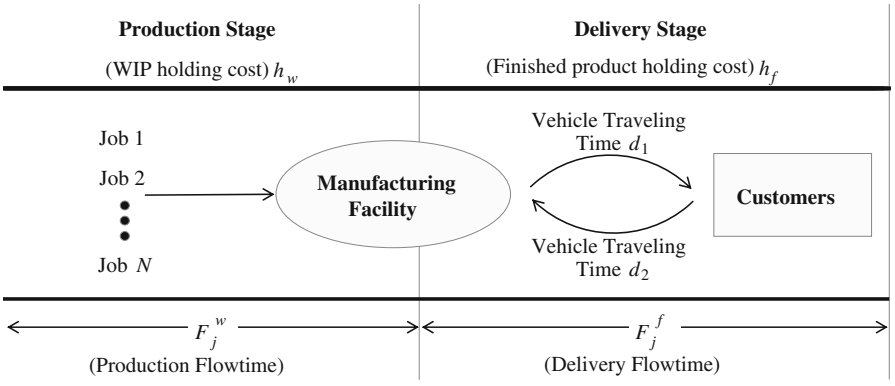


**Fig. 1.** Graphical representation of the proposed problem.