Expert Systems with Applications 38 (2011) 1909–1916

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Decision-making in a single-period inventory environment with fuzzy demand

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ARTICLE INFO

Keywords:
Newsboy
Achievable capacity index
Decision-making
Fuzzy sets
Fuzzy hypothesis testing

ABSTRACT

This paper first defines the profitability to be the probability of achieving a target profit under the optimal ordering policy, and introduces a new index (achievable capacity index; I_A) which can briefly analyze the profitability for newsboy-type product with normally distributed demand. Note that since the level of profitability depends on the demand mean μ and the demand standard deviation σ if the related costs, selling price, and target profit are given, the index I_A is a function of μ and σ. Then, we assess level performance which examines if the profitability meets designated requirement. The results can determine whether the product is still desirable to order/manufacture. However, μ and σ are always unknown, and the demand quantity is common to be imprecise, especially for new product. To tackle these problems, a constructive approach combining the vector of fuzzy numbers is introduced to establish the membership function of the fuzzy estimator of I_A. Furthermore, a three-decision testing rule and step-by-step procedure are developed to assess level performance based on fuzzy critical values and fuzzy *p*-values.

1. Introduction

The classical newsboy problem (single-period problem) deals with the purchasing inventory problem for short shelf-life products with the uncertainty of demand. For such problems, the managers should determine ordering quantity at the beginning of each period. Products cannot be sold in the next period and need additional cost (excess cost) to dispose it if the ordering quantity exceeds actual demand. Therefore, the determination of the ordering quantity is critical in the classical newsboy problem. Several extensions to the newsboy problem have been proposed and solved in the literature. Among those extensions are alternative objective functions such as minimizing the expected cost (Nahmias, 1993), maximizing the expected profit (Khouja, 1995), maximizing the expected utility (Ismail & Louderback, 1979; Lau, 1980), and maximizing the probability of achieving a target profit (Ismail & Louderback, 1979; Khouja, 1996; Lau, 1980; Sankarasubramanian & Kumaraswamy, 1983; Shih, 1979). In fact, these maximum and minimum values can be adopted to measure product's capacity. For example, the maximum expected profit and maximum probability of achieving the target profit can measure product's profitability.

In this paper, we consider the newsboy-type product with normally distributed demand and assume that the profitability is defined to be the probability of achieving a target profit under the optimal ordering policy. Furthermore, in order to simplify the calculation, we develop a new index, which has a simple form and can correspond to the profitability, and so-called "achievable capacity index (ACI)", and be denoted by I_A. Note that since the level of profitability depends on the demand mean μ and the demand standard deviation σ if the related costs, selling price, and target profit are given, the index I_A is a function of μ and σ. Then, we assess level performance which examines if the profitability meets designated requirement. However, μ and σ are always unknown. To tackle this problem, one should collect the historical data of demand, and then implement the following hypothesis testing, $H_0 : I_A \leqslant C$ versus $H_1 : I_A > C$, where C is a designated requirement. Critical value of the test must be calculated to determine the results. The results can determine whether the product is still desirable to order/manufacture. But in practice, especially for new product, the demand quantity is difficult to acquire due to lack of information and historical data. In this case, the demand quantity is approximately specified based on the experience. Some papers have dealt with this case by applying fuzzy theory. Petrovic, Petrovic, and Vujosevic (1996) first proposed a newsboy-type problem with discrete fuzzy demand. Dutta, Chakraborty, and Roy (2007) studied the newsboy problem with reordering opportunities under fuzzy demand. Zhen and Xiaoyu (2006) considered the multi-product newsboy problem with fuzzy demands under budget constraint. Kao and Hsu (2002) compared the area of fuzzy numbers to obtain the optimal order quantity. To the best of our knowledge, no researchers have investigated the fuzzy hypothesis testing for assessing level performance. In this study, we first use a

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doi:10.1016/j.eswa.2010.07.123

new approach in fuzzy statistics to estimate the demand mean and variance parameters of normal distribution (Buckley, 2004, 2005a, 2005b). Then, a general method combining the vector of fuzzy numbers of sample mean $\tilde{\bar{x}}$, and sample variance $\tilde{s}^2$ is proposed to derive the membership function of the fuzzy estimator of I_A. Furthermore, a three-decision testing rule for assessing level performance according to two different criteria, critical value and fuzzy p-value are proposed. Based on the test, we develop a step-by-step procedure for managers to use so that decisions made in examining the profitability are more reliable. The rest of the paper is organized as follows. In the next section, we calculate the profitability, and develop a new index I_A to correspond profitability. Section 3 discusses the statistical properties of estimation for I_A based on crisp data. In Section 4, we present some basic definitions, notations of fuzzy sets and the α-cuts of fuzzy estimation for I_A. Section 5 deals with implementing fuzzy hypothesis testing for assessing level performance. Following critical value and fuzzy p-value, decision rules and testing procedures are developed. In Section 6, a numerical example is discussed to illustrate the procedure of solving the problem. Some conclusions are given in the final section.

2. Profitability and achievable capacity index I_A

The total profit function, Z, in the newsboy model depends on the demand quantity D and ordering quantity Q, and is formulated as

$$Z = \begin{cases} c_pD - c_e(Q-D) = (c_p+c_e)D - c_eQ, & 0 \leqslant D \leqslant Q, \\ c_pQ - c_s(D-Q) = -c_sD + (c_p+c_s)Q, & Q < D < \infty, \end{cases}$$

where

- c_p the net profit per unit (selling price per unit minus purchasing cost per unit),
- c_e the excess cost per unit (purchasing cost per unit plus disposal cost per unit; $c_p > c_e > 0$),
- c_s the shortage cost per unit ($c_p > c_s > 0$),
- k the target profit which is set according to the product property and the sales experience.

Note that if $c_pQ < k$, then the profit impossibly achieve the target profit, even the demand is large enough. Therefore, the order quantity should be at least k/c_p. For $Q \geqslant k/c_p$, Z increases for $0 \leqslant D \leqslant Q$ and decreases for $D \geqslant Q$, and has a maximum at point $D = Q$. The maximum value of Z is equal and higher than k, i.e., $Z = c_pD = c_pQ \geqslant k$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$. So the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_eQ + k}{c_p + c_e} \quad \text{and} \quad UAL(Q) = \frac{(c_p + c_s)Q - k}{c_s},$$

are the lower and upper achievable limits, respectively, and both are the functions of Q.

Under the assumption that the demand is normally distributed, the probability of achieving the target profit is

$$\Pr[Z \geqslant k] = \Phi\left(\frac{UAL(Q)-\mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q)-\mu}{\sigma}\right) = \Phi\left(\frac{d(Q)+m(Q)-\mu}{\sigma}\right) - \Phi\left(\frac{-d(Q)+m(Q)-\mu}{\sigma}\right), \quad (1)$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution, $d(Q) = [UAL(Q) - LAL(Q)]/2$ is the half-length of the achievable interval $[LAL(Q), UAL(Q)]$, and $m(Q) = [UAL(Q) + LAL(Q)]/2$ is the midpoint between the lower and upper achievable limits. Since the necessary condition for maximizing $\Pr[Z \geqslant k]$ is $\mathrm{d}\Pr[Z \geqslant k]/\mathrm{d}Q = 0$, we have

$$\mu = m(Q) - \frac{\omega\sigma^2}{2d(Q)}, \quad (2)$$

where $\omega = \ln[1 + c_pA/c_sc_e] > 0$ and $A = c_p + c_e + c_s$. For $Q \geqslant k/c_p$, the optimal ordering quantity can be obtained by solving Eq. (2), i.e.:

$$Q^* = \frac{k}{c_p} + \frac{c_s(c_p+c_e)(c_p\mu - k)}{c_p(c_pA + 2c_ec_s)} + \sqrt{\left[\frac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}\right]^2 + \frac{2c_s^2(c_p+c_e)^2\omega\sigma^2}{c_pA(c_pA+2c_ec_s)}} > \frac{k}{c_p}. \quad (3)$$

In addition, the sufficient condition is also calculated as follows:

$$\left.\frac{\mathrm{d}^2\Pr[Z \geqslant k]}{\mathrm{d}Q^2}\right|_{Q=Q^*} = -\frac{(c_p+c_s)e^{-\frac{1}{2}\left(\frac{UAL(Q^*)-\mu}{\sigma}\right)^2}}{\sqrt{2\pi}\sigma^3c_s^2(c_p+c_e)}\left[d(Q^*)(c_pA+2c_ec_s) + \frac{c_pA\omega\sigma^2}{2d(Q^*)}\right] < 0. \quad (4)$$

As a result, it leads to the conclusion that Q^* is the optimal ordering quantity that maximizes the probability of achieving the target profit. By using Eq. (2) and substituting Eq. (3) into Eq. (1), the profitability, $\mathbb{P}$, can be obtained as follows:

$$\mathbb{P} = \Phi\left(\frac{d(Q^*)}{\sigma} + \frac{\omega\sigma}{2d(Q^*)}\right) - \Phi\left(-\frac{d(Q^*)}{\sigma} + \frac{\omega\sigma}{2d(Q^*)}\right). \quad (5)$$

2.1. Achievable capacity index I_A

We develop a new index to express the product's profitability. It is defined as:

$$I_A = \frac{d(Q^*)}{\sigma} = \frac{M(c_p\mu - k)}{\sigma} + \sqrt{\left[\frac{M(c_p\mu-k)}{\sigma}\right]^2 + c_pM\omega}, \quad (6)$$

where $M = A/2(c_pA + 2c_ec_s)$, and call it "achievable capacity index". The numerator of I_A provides the half demand range over which the total profit will achieve the target profit under the optimal order quantity. The denominator gives demand standard deviation. Obviously, it is desirable to have a I_A as large as possible. From the Eq. (5), $\mathbb{P}$ can be rewritten as follows:

$$\mathbb{P} = \Phi\left(I_A + \frac{\omega}{2I_A}\right) - \Phi\left(-I_A + \frac{\omega}{2I_A}\right). \quad (7)$$

It is easy to see that $\mathbb{P}$ is the function of I_A. Taking the first-order derivative of $\mathbb{P}$ with respect to I_A, we obtain

$$\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}I_A} = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(I_A + \frac{\omega}{2I_A}\right)^2}\left[\frac{\omega}{2I_A^2}(e^{\omega}-1) + e^{\omega} + 1\right] > 0. \quad (8)$$

Table 1
The profitability for I_A = 0.5(0.5)4.0 and ω = 0.5(0.5)5.0.

ω	I_A 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.5	0.3413	0.6677	0.8610	0.9528	0.9871	0.9972	0.9995	0.9999
1.0	0.2417	0.6247	0.8450	0.9477	0.9858	0.9969	0.9995	0.9999
1.5	0.1359	0.5586	0.8186	0.9391	0.9835	0.9964	0.9994	0.9999
2.0	0.0606	0.4773	0.7825	0.9270	0.9803	0.9957	0.9993	0.9999
2.5	0.0214	0.3891	0.7377	0.9111	0.9759	0.9948	0.9991	0.9999
3.0	0.0060	0.3023	0.6853	0.8914	0.9703	0.9936	0.9989	0.9998
3.5	0.0013	0.2236	0.6267	0.8677	0.9634	0.9920	0.9986	0.9998
4.0	0.0002	0.1573	0.5639	0.8400	0.9550	0.9901	0.9983	0.9998
4.5	0.0000	0.1051	0.4987	0.8083	0.9449	0.9877	0.9978	0.9997
5.0	0.0000	0.0666	0.4330	0.7728	0.9330	0.9848	0.9973	0.9996