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Exact and approximate calculation of the cycle service level in periodic review inventory policies

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ABSTRACT

The parameters of stock policies are usually determined to minimize costs while satisfying a target service level. In a periodic review policy the time between reviews can be selected to minimize costs while the order-up-to-level is based on the fulfilment of a target service level. Generally, the calculation of this service measurement is obtained using approximations based on an additional hypothesis related to the demand pattern. Previous research has shown that there is a substantial difference between exact and approximate calculations in some general circumstances, so in these cases the service level is not accomplished or the stock level is overestimated. Although an exact calculation of *CSL* was developed in previous work, the computational effort required to apply it in practical environments leads to the proposal of two approximate methods (*PI* and *PII*) that, with the classic approximation, are analysed and evaluated in this paper. This analysis points out the risks of using the *classic* approximation and leads one to suggest *PII* as the most suitable and accurate enough procedure to compute the *CSL* straightforwardly in practice. Additionally, a heuristic approach based on *PII* is proposed to accept or reject an inventory policy in terms of fulfilling a given target *CSL*. This paper focuses on uncorrelated, discrete and stationary demand with a known distribution pattern and without backlog.

1. Introduction and literature review

The estimation of the cycle service level, *CSL*, in the traditional periodic review, order-up-to-level (R, S) system is based on the assumptions detailed by Silver et al. (1998) that are inappropriate when managing intermittent and slow movement demand items. It is especially relevant for the purpose of this paper that one of the main underlying assumptions related to the (R, S) formulation is the negligible chance of no demand between two consecutive reviews. This is because in an intermittent demand context: (i) the probability of no demand when the physical stock is equal to zero is not negligible and (ii) there is a chance of no demand during the replenishment cycle. However, despite the violation of this hypothesis the traditional (R, S) policy is applied in an intermittent demand context such as the model suggested by Syntetos and Boylan (2006) which is based on Sani and Kingsman (1997), where demand during the lead time is modelled using the negative binomial distribution or the model proposed by Leven and Segerstedt (2004), which uses the Erlang distribution.

According to Cardós et al. (2006), when the demand pattern does not meet the hypotheses mentioned above, the procedure used to estimate the service level is only approximate and eventually may show large deviations. The *CSL* is defined by Chopra and Meindl (2004) as *"the probability of not stocking out in a replenishment cycle"*. Silver et al. (1998) defines the stockout as "*an occasion when the available physical stock drops to the zero level*". Therefore, *CSL* is defined as the fraction of cycles in which the physical stock does not drop to the zero level. Surprisingly, this definition, called classic in this paper, does not take into account demand fulfilment. Furthermore, if the system is managed using a (R, S) policy, the classic definition leads one to consider that the *CSL* is equal to one if there is no demand during the replenishment cycle. For these reasons Cardós et al. (2006) suggest a more standard and useful definition capable of dealing with any type of stationary, discrete and independent and identically distributed (*i.i.d.*) demand pattern as the fraction of cycles in which non-zero demand is completely met by the physical stock. In this definition, the fulfilment of demand is explicitly considered and it works properly even if there is no demand during the replenishment cycle. According to this standard definition, with the physical stock at the beginning of the replenishment cycle being z_0 and D_R the demand during this cycle, the exact *CSL* value is

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calculated as

$$CSL(z_0) = P(D_R \le z_0 | D_R > 0) = \frac{P(0 < D_R \le z_0)}{P(D_R > 0)} \quad (1)$$

However, to compute the exact *CSL* when the physical stock at the beginning of the replenishment cycle is not known a priori is quite complex, requiring the availability of appropriate tools as well as a sound mathematical background and eventually it may also be time consuming. Therefore, the exact method to compute the *CSL* is not an appropriate procedure to be widely used in a business context. This fact justifies the twofold objectives of this paper. Firstly, it points out the risks of using the *classic CSL* definition and secondly, this paper proposes two new approximations, *PI* and *PII*, in order to provide a suitable and accurate approximate procedure to compute the *CSL* but which is computationally simple enough to be used straightforwardly in practice.

This paper is organized as follows. Section 2 is dedicated to describing the derivation of the exact method to compute the *CSL* for the (R, S) policy derived by Cardós et al. (2006), since this is the starting point of this paper. Section 3 proposes approximations *PI*, *PII* and the *classic* one. Section 4 presents a comparison between the exact and the approximate methods to compute the *CSL* and the discussion of the results from it. Section 5 compares the performance of the approximations based on their ability to provide the exact inventory policy and shows the risks involved. Finally, conclusions and further research are briefly pointed out in Section 6.

2. Exact calculation of the cycle service level in a periodic review policy

In general, periodic review policies place replenishment orders every R fixed time periods to reach the order up to level S. The replenishment order is received L time periods after being launched. Fig. 1 shows an example of the evolution of the physical stock in a periodic review system. The notation used in it and in the rest of the paper is:

S = order up to level,
R = review period and replenishment cycle corresponding to the time between two consecutive deliveries,
L = lead time for the replenishment order,
z_t = physical stock in time t from the first reception,
D_t = accumulated demand during t consecutive periods,
X^+ = maximum $\{X, 0\}$ for any expression X,
$f_t(\cdot)$ = probability density function of demand in t,
$F_t(\cdot)$ = cumulative distribution function of demand during t periods.

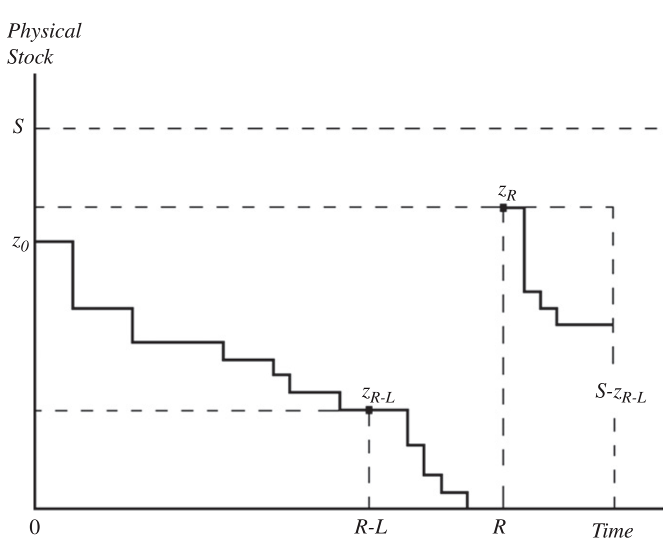

Fig. 1. Example of stock evolution in a periodic review system.

Cardós et al. (2006) consider the following assumptions to derive the exact *CSL* method: (i) L is constant; (ii) backordering is not allowed and therefore $L < R$; (iii) the replenishment order is added to the inventory at the end of the period in which the order is received; (iv) demand during a period is fulfilled with the inventory at the beginning of that period and (v) the demand process is assumed to be stationary with a known, discrete and *i.i.d.* distribution function. Then, the authors consider that the stock balance at R–L is

$$z_{R-L} = [z_0 - D_{R-L}]^+ \quad (2)$$

therefore

$$P(z_{R-L} = j) = \sum_{i=1}^{S} P(z_{R-L} = j | z_0 = i) P(z_0 = i) \quad (3)$$

expressing (3) as matrixes

$$P(\overline{z_{R-L}}) = P(\overline{z_0}) \cdot \overline{\overline{M_{R-L}}} \quad (4)$$

where

$$\overline{\overline{M_{R-L}}} = [m_{ji}] \quad (5)$$

and according to (2)

$$m_{ij} = P(j = [i - D_{R-L}]^+) = \begin{cases} P(D_{R-L} \ge i) = 1 - F_{R-L}(i-1) & j = 0 \\ P(D_{R-L} = i - j) = f_{R-L}(i-j) & j > 0 \end{cases} \quad (6)$$

From (6) and (4) it is easy to obtain the probability of every stock level at R–L. Following the same reasoning, the stock balance at R can be expressed as

$$z_R = [z_{R-L} - D_L]^+ + S - z_{R-L} \quad (7)$$

Analogously,

$$P(\overline{z_R}) = P(\overline{z_{R-L}}) \cdot \overline{\overline{M_L}} \quad (8)$$

where

$$\overline{\overline{M_L}} = [m_{kj}] \quad (9)$$

Hence

$$m_{kj} = P(k = [j - D_L]^+ + S - j) = \begin{cases} 0, & k + j - S < 0 \\ 1 - F_L(j-1), & k + j - S = 0 \\ f_L(S-k), & k + j - S > 0 \end{cases} \quad (10)$$

Thus, from expressions (4) and (8)

$$P(\overline{z_R}) = P(\overline{z_{R-L}}) \cdot \overline{\overline{M_L}} = P(\overline{z_0}) \cdot \overline{\overline{M_{R-L}}} \cdot \overline{\overline{M_L}} = P(\overline{z_0}) \cdot \overline{\overline{M_R}} \quad (11)$$

where $\overline{\overline{M_R}} = \overline{\overline{M_{R-L}}} \cdot \overline{\overline{M_L}}$ is defined as the transition matrix between the inventory levels from the beginning of the replenishment cycle to its end. Therefore, it can be deduced that

$$P(\overline{z_{mR}}) = P(\overline{z_0}) \cdot \overline{\overline{M}}_R^m \quad (12)$$

And if the powers of the transition matrix converge to $\overline{\overline{M}}$ then

$$\lim_{m \to \infty} P(\overline{z_{mR}}) = P(\overline{z_0}) \cdot \overline{\overline{M}} \quad (13)$$

where $P(\overline{z_0})$ is equal to one of the equal rows of the transition matrix.

As a consequence, the *CSL* can be calculated in general as

$$CSL = \sum_{z_0=0}^{S} P(z_0) \cdot CSL(z_0) = \sum_{z_0=0}^{S} P(z_0) \cdot \frac{F_R(z_0) - F_R(0)}{1 - F_R(0)} \quad (14)$$