Int. J. Production Economics 131 (2011) 727–735

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimization of a vendor managed inventory supply chain with guaranteed stability and robustness

Darya Kastsian, Martin Mönnigmann *

Automatic Control and Systems Theory, Ruhr-Universität Bochum, 44801 Bochum, Germany

A R T I C L E I N F O

Article history:
Received 4 August 2010
Accepted 19 February 2011
Available online 24 February 2011

Keywords:
Supply chain optimization
Parametric uncertainty
Robust stability
Inventory control
Normal vector method

A B S T R A C T

In this work we address the steady state optimization of a supply chain model that belongs to the class of vendor managed inventory, automatic pipeline, inventory and order based production control systems (VMI-APIOBPCS). The supply chain is optimized with the so-called normal vector method, which has specifically been developed for the economic optimization of uncertain dynamical systems with constraints on dynamics. We demonstrate that the normal vector method provides robust optimal points of operation for a number of scenarios. Since the method strictly distinguishes economic optimality, which is treated as the optimization objective, from dynamical requirements, which are incorporate by appropriate constraints, it provides a measure for the cost of stability and robustness as a desired side-effect.

1. Introduction

Supply chain dynamics have been investigated with a variety of methods. Wang et al. (2009) review recent publications and group the existing methods into three classes, namely (i) those based on control theory, (ii) those based on behavioral science, and (iii) practitioner approaches. Approaches that belong to the first class (e.g., Disney and Towill, 2002a; Lin et al., 2004) use transfer functions found with Laplace- or z-transforms to analyze the stability of continuous time and discrete supply chain models, respectively. Investigations that fall into category (ii) are usually used for models which describe business games. For this class Lyapunov exponents (e.g., Hwarng and Xie, 2008; Larsen et al., 1999) can be used to characterize different types of dynamic behavior. The third category groups those approaches that are based on extensive simulation studies (e.g., Nagatani and Helbing, 2004; Sahin et al., 2008).

Supply chain dynamics play a crucial role in optimization. Unfortunately, economic optimality and optimal dynamical properties can often not be attained simultaneously. This is not surprising, since any non-trivial optimization pushes the optimized system to some or all of its operational boundaries, among them possibly its stability boundaries. There exist various approaches to considering dynamical properties when optimizing supply chain operation. Sarmiento et al. (2007) minimize the area under the curves of step responses in order to achieve stable operation. Similarly, Disney et al. (2000) and Disney and Towill (2002b) minimize the area under the step response of the actual inventory. In addition, these authors minimize a measure of the Bullwhip effect.

If optimization results in a mode of operation that lies on, or close to, any operational or stability boundaries, it is important to take model uncertainty into account. For an optimal point on, or close to, any such boundary, even a slight change in parameters may result in a violation of that boundary. Model uncertainty is particularly important, since parameters of supply chain models are quite imprecise. Following Disney et al. (2000) we consider uncertainties of up to 25% in the present paper.

Model uncertainty has been accounted for by several authors. Stochastic programming (e.g., Azaron et al., 2008), fuzzy programming (e.g., Mitra et al., 2009; Lin et al., 2010), and probabilistic programming (e.g., You and Grossmann, 2008) are three popular approaches to handle uncertainties in supply chain optimization (see Mitra et al., 2008 for a summary of recent literature). Note that these approaches do not guarantee stability or other dynamical properties of the resulting optimal points, however.

In the present paper we apply the so-called normal vector method (Mönnigmann and Marquardt, 2002). The normal vector method was developed for solving optimization problems in which stability or related dynamical properties have to be guaranteed for systems with uncertain parameters. In contrast to the methods used by Disney et al. (2000), Disney and Towill (2002b) and Sarmiento et al. (2007), the normal vector method does not treat the dynamical properties as an optimization

* Corresponding author. Tel.: +49 234 3224060; fax: +49 234 3214155.
E-mail addresses: darya.kastsian@rub.de (D. Kastsian), martin.moennigmann@rub.de (M. Mönnigmann).

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doi:10.1016/j.ijpe.2011.02.022

objective, but dynamical properties become constraints of a constrained optimization problem. Note that this implies a separation of economic optimality, which is considered in the cost function, from required dynamical properties, which are treated in the constraints. In particular, there is no need for combining economic optimality and dynamical requirements into a multi-objective cost function.

The normal vector method has originally been designed for the optimization of continuous-time systems that can be modeled by sets of parametrically uncertain ordinary differential equations (Gerhard et al., 2008; Mönnigmann and Marquardt, 2002, 2003, 2005; Mönnigmann et al., 2007). It has recently been extended to discrete-time systems with uncertain parameters (Kastsian and Mönnigmann, 2008, 2010). In fact, supply chain optimization served as an example in these publications. The separation of economic optimization from dynamical constraints for robustness is achieved for the first time in the present paper, however.

The paper is organized as follows. Section 2 introduces the model of the considered supply chain. This model belongs to a class of systems referred to as vendor managed inventory, automatic pipeline, inventory and order based production control system (VMI-APIOBPCS for short) (Disney and Towill, 2002a,b). Section 3 describes the cost function. Section 4 presents the result of an optimization without any constraints on the dynamics, which serves as a reference in the remaining sections. This is followed by a discussion of the stability of the supply chain in Section 5. The normal vector method is introduced in Section 6 and applied to the supply chain in Section 7. A summary and conclusion are given in Sections 8 and 9, respectively.

2. Outline of the VMI-APIOBPCS supply chain model

The VMI-APIOBPCS is a production control model for a single stage and single product manufacturing system. In supply chains without VMI, each echelon is responsible for its inventory control and orders. In contrast, the information on inventory or demand is shared between customers and suppliers in VMI supply chains. The VMI relationship is known to reduce the Bullwhip effect (Disney and Towill, 2003).

The structure of the VMI-APIOBPCS model is shown in Fig. 1. The optimal safety stock (target inventory, TINV for short) and the demand signal (customer orders per unit of time, CONS) are the input variables, or reference signals, of the closed-loop system. The actual inventory level of produced goods (AINV) and work in progress (WIP) are the output variables. Two feedback loops exist for the order rate (ORATE). The AINV is fed back and compared to the TINV to give the error in inventory (EINV). Secondly, the WIP is fed back and compared to the desired work in progress (DWIP) to result in the error in work in progress (EWIP). Completion rate (COMRATE) is the delayed function of the ORATE. Note that several quantities are smoothened in the system to average out fluctuations. Virtual consumption (VCON) is created to share information on CONS between customers and suppliers. The smoothed average of VCON (AVCON) is used to estimate future sales.

The VMI-APIOBPCS model can concisely be stated as a set of discrete-time difference equations of the general form

$$x_k = f(x_{k-1}, p), \quad x_0 = x(0), \tag{1}$$

where x and p denote n_x- and n_p-dimensional vectors of state variables and model parameters, respectively. The function f is assumed to be sufficiently smooth with respect to x and p for all $k \in \mathbb{N}$. The discrete-time difference equations for the particular VMI-APIOBPCS treated here are summarized in Appendix A. For a more detailed description we refer the reader to Towill (2002a,b).

The following quantities are fixed throughout the article. The unit of time is chosen to be 1 week. The production delay Tp and the constant $T\overline{p}$ that affect the calculation of COMRATE and DWIP, respectively, are set to four time periods, i.e. to 4 weeks. TINV is set to zero. Finally, the parameter G, which sets the customer service level, is fixed to 8. All settings are adopted from Disney and Towill (2002b). A sensitivity analysis with respect to parameters Tp and G is given in Appendix C.

The VMI-APIOBPCS described above has four uncertain parameters, namely Tw, Ti, Ta, and Tq (i.e. $n_p = 4$). The parameters Tw and Ti describe how strongly the pipeline feedback and the inventory feedback affect the order rate (cf. Fig. 1). Ta denotes how quickly the demand is tracked for the forecasting of sales. Tq is the parameter which affects the sales signal when re-order points are generated. These four parameters are subject to optimization.

3. Optimization objective for the VMI-APIOBPCS

In this section we will define the cost function that is to be minimized in the VMI-APIOBPCS optimization. We assume that the cost is linearly related to the variance ratios of the order rates

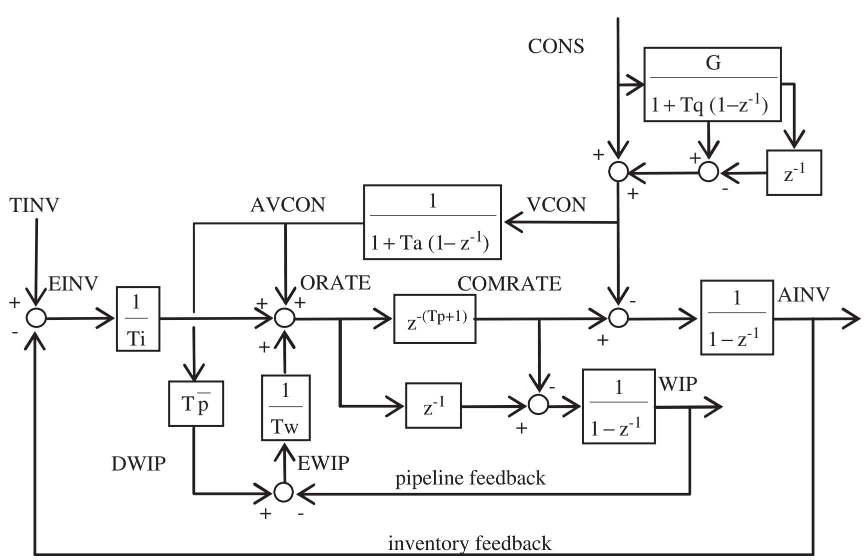

Fig. 1. Block diagram of the VMI-APIOBPCS (reproduced after Disney and Towill, 2002a,b).