Int. J. Production Economics 131 (2011) 663–673

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An inventory and order admission control policy for production systems with two customer classes

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ARTICLE INFO

Article history:
Received 1 April 2009
Accepted 9 February 2011
Available online 19 February 2011

Keywords:
Inventory rationing
Admission control
Production control
Markov chains

ABSTRACT

In this paper we examine a Markovian single-stage system producing a single item to satisfy demand of two different customer classes. A simple threshold type heuristic policy is proposed for the joint control of inventories and backorders. Explicit forms of the steady-state probabilities under this policy are derived and used to assess the average profit rate of the system and determine the optimal control parameters. Certain properties of the average profit rate are established and used to develop computationally efficient algorithms for finding the optimal control parameter values. Numerical results show that the proposed policy is a very good approximation of the optimal policy and outperforms other commonly used policies.

1. Introduction

We consider a single-stage stochastic manufacturing system producing a single product type to satisfy random demand from two classes of customers. The objective is to decide when to stop or start producing (production control), whether to satisfy an incoming order immediately from stock (inventory rationing control), or backorder it, or even reject it (order admission control), so as to maximize profit from sales minus inventory and backlog costs.

The problem of inventory rationing, that is, satisfying different customers from a common stock, has extensively been studied in the last decades (see, e.g., Ha, 1997a,b, 2000; de Verikourt et al., 2002; Lee and Hong, 2003; Melchiors, 2003; Gayon et al., 2009). Such problems arise in many occasions in the context of production systems. One example is delayed product differentiation. In that case a common stock of standard items is held and customer demands are satisfied through a rapid differentiation operation. Another example is manufacturers operating both as wholesalers and retailers. A similar situation is when a part supplier has to satisfy demand from both original equipment manufacturers and the so-called after-market. This is the case in the automobile industry, where a part supplier sells his products to manufacturers producing new vehicles, and to repair shops as well. Most of the work in this area assumes that the demand during stockout periods is fully backordered (Ha, 1997b; de Verikourt et al., 2002), or completely lost (Ha, 1997a, 2000). This is also the case in the production systems control literature in general. Most of the work reported in the literature deals with situations where the expected aggregate demand is assumed to be much less than the available capacity. Typically, total demand is assumed to be in the range of 80–85% (or less) of the available capacity. Recent empirical data, however, suggests that many firms face total expected demand much higher than that (Sridharan, 1998). Fransoo et al. (1995) present the example of a glass-containers manufacturer in The Netherlands, where the actual aggregate demand level has been greater than 95% of the available capacity. In other cases, such as high-fashion apparel industry, printing shops, and low volume component manufacturing shops, the total demand is even greater than the available capacity as described in Balakrishnan et al. (1996). One may argue that when demand is greater than capacity, the firm should expand capacity to solve the problem. Although this could be a viable long-term option, it is not a feasible option in the short term. Furthermore capacity expansion is often precluded as an economically viable option for some firms due to the high fixed-to-variable cost ratio (Wheelwright and Hayes, 1985) and the need for a highly skilled labor force. Thus, other alternatives need to be considered. Ensuring coordination between marketing and manufacturing functions appears to be an attractive alternative in the above situations. The objective, then, is to efficiently allocate the available capacity between competing requirements and to manage demand, so as to maximize profit and delivery reliability. Recently the problems of joint production and admission control have attracted some attention (see, e.g., Caldentey, 2001; Song, 2006; Ioannidis et al., 2008). For the problem of inventory rationing, Benjaafar et al. (2007) have incorporated order admission decisions for two priority classes of customers and proved that the optimal policy is characterized by monotone switching

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doi:10.1016/j.ijpe.2011.02.011

curves. Another interesting result of their work is that order admission control significantly improves the performance of the system.

In a monotone switching policy, decisions as to whether produce or stop producing, accept or reject an incoming order of high (class 1) or low (class 2) priority, and fill all orders from stock or reserve stock only for high priority (class 1) customers change dynamically with the state of the system. For example, when a class 2 order arrives during a stockout period, the fewer the number of pending orders, the easier it is to make the decision to accept the new order (monotonicity). However, the threshold that separates the state space into a region of acceptance decisions and a region of rejection decisions depends monotonically on the specific backlogs of class 1 and class 2 orders encountered upon the arrival of the new order. Unfortunately, such dependencies are nonlinear and do not have analytical expressions. An approach to this problem involves the use of threshold type policies, which are switching policies with fixed thresholds and linear dependencies. For the example given above, a threshold policy would be to accept an arriving class 2 order only when the total backlog (linear function of the individual ones) is below a prespecified threshold. Threshold policies are in general easier to both optimize and implement. However, the key condition under which such a policy can be useful for practical purposes is when it achieves a good performance relative to the performance of the optimal policy.

Our work makes three contributions:

(1) In this paper, we propose a simple threshold type policy for a two-class system in which the production, service, and backordering decisions are integrated as in Benjaafar et al. (2007). The proposed policy has four control parameters. The first parameter determines when to switch off and on the production facility depending on the current inventory, the second one forces the system to maintain an inventory rationing level for high-priority customers, and the other two parameters determine when to reject incoming orders depending on the backorders of both customer classes.
(2) A new modeling approach is presented for the assessment of the steady state probabilities and the average profit rate of the system.
(3) Apart from the analysis approach, this paper presents a number of structural properties of the profit rate function, which give rise to efficient optimization algorithms to track down the optimal values of the control parameters so as to maximize the overall profit rate of the system in equilibrium.

The rest of the paper is organized as follows. In Section 2, the problem is presented and formulated as a finite state Markov chain. In Section 3 steady-state probabilities of the Markov chain are derived and used to compute the average profit rate. Structural properties of the average profit rate function are presented, which are used for the construction of a computationally efficient algorithm for the assessment of the optimal control parameters. In Section 4, the proposed policy is compared numerically with the optimal policy and other commonly used backordering policies. The results indicate that the proposed policy is a very good approximation of the optimal policy and superior to the other backordering policies.

2. Problem description

Consider a production facility that produces a single product. Customers arrive at random times and each customer requests one unit of product. Two customer classes denoted 1 and 2 arrive according to Poisson processes with rates λ_1 and λ_2, respectively. When orders of both customer classes are pending, the system gives priority to class 1 customers. Processing times are independent, with exponential random variables with mean $1/\mu$. Finished items are stored in an output buffer.

The system incurs holding costs when products are made to stock in anticipation of future demand, backordering costs when orders are filled with delay, and, possibly, costs of lost sales for rejected customer orders. The overall system performance is defined as the long-run rate of profit from sales less the costs outlined above. Next, we describe a control policy that strikes a balance among the components of the overall performance measure while prioritizing class 1 customers. This policy is characterized by three types of decision: production control, admission control, and inventory rationing:

(i) Production control uses a safety stock to protect the system against stockouts but also against excessive inventories. The production facility produces to stock as long as the number of finished items is less than a certain threshold s, called the *base stock*, and stops when the inventory level reaches s. It is assumed that no cost or delay is incurred when the facility is switched on and off.
(ii) The system avoids excessive backlogs by employing a class-dependent admission control policy. A class 1 order arriving during a stockout period is accepted if the number of class 1 pending orders is less than a *base backlog* c_1 and rejected otherwise; if there is stock available, the order is accepted and satisfied immediately. The admission of class 2 customers depends on the *backlog position*, which is defined as the total number of pending orders of both classes minus the inventory level. An arriving class 2 order is accepted when the backlog position is less than a *base backlog* threshold c_2 and rejected otherwise.
(iii) Accepted orders of class 1 have priority and are served immediately if stock is available. Accepted class 2 orders are satisfied only after all class 1 orders are filled and a sufficient amount of inventory is built up in the system. Specifically, if pending orders of both customer classes are present and the system produces one item, then this item will fill a class 1 order. This continues until all class 1 orders (those originally pending and those that have arrived in the meantime) are filled and the system is left only with class 2 orders pending. Thereafter, the system starts to produce to stock and keeps all accepted class 2 orders pending until the inventory reaches a certain *stock rationing* level σ, $\sigma \geq 0$. This stock is set aside to satisfy future class 1 orders without delay. Finally, class 2 pending orders are satisfied one at a time only when the production facility produces items in excess of σ.

The above policy will be referred to as the *dependent double base backlog* policy (DDBB). DDBB is a fixed-parameter, make-to-stock policy that includes some commonly used policies as special cases like complete backordering (CB) policies that have $c_1 = c_2 = \infty$.

The net profit rate of the system is a function of the four control parameters s, σ, c_1, and c_2. We define the following financial parameters:

α_i unit profit from selling a product to customers of class i (selling price less cost of purchasing raw material and processing per item),
β_i cost, if any, per lost sale (due to, e.g., customer loss of goodwill, contractual penalties, etc.)