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# A network approach to modeling the multi-echelon spare-part inventory system with backorders and interval-valued demand

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ABSTRACT

A multi-echelon inventory system implies the existence of a hierarchy of stocking locations, and the dependence and interaction between them. We consider a multi-echelon, spare-part inventory management problem with outsourcing and backordering. The problem is characterized by deterministic repair time/cost, and supply and demand that lie within prescribed intervals and that vary over time. The objective is to minimize the total inventory and transportation costs. We develop a network model for problem analysis and present a network flow algorithm for solving the problem. We prove that the Wagner–Whitin property, known for the lot-sizing problem, can be extended to the spare-part inventory management problem under study.



## 1. Introduction

For many expensive technological systems, such as computer systems, medical equipment, and military defence systems, high availability of spare parts is essential. As the spare parts can be expensive, it is often more economical to repair them than to replace them. Timely supply of spare parts can be achieved by holding sufficient stocks and/or minimizing the repair work. A trade-off between spare-part inventories and repair facilities is achieved by increasing spare-part inventories and decreasing the repair capacity, and vice versa. Thus an optimization problem concerning the optimal investment allocation to both spare-part inventories and repair capacity arises.

Inventory systems where units that fail are repaired at a repair shop rather than discarded are called *repairable-item inventory systems*. A repairable spare-part network implies the existence of locations where spare parts are stocked and that there are facilities to repair failed items. In this paper we consider a multi-echelon inventory system with several operational sites (the bases) and two supply modes, namely an external supplier and a repair shop (called the *depot*). A failed item is sent from its base to the repair shop and the repaired item is returned to the originating base. We investigate the repairable-item inventory management problem from the perspective of network analysis, while past studies have taken disparate approaches to treat the problem (see, e.g., the surveys by Axsäter (1990), Díaz and Fu (1995), Kennedy et al. (2002), Minner (2003), and Sleptchenko et al. (2005), and the numerous references contained in them).

There is extensive literature that addresses spare-part inventory systems with random demand under different assumptions on the data probability distributions. Hausman and Scudder (1982), Pyke (1990), Verrijdt et al. (1998), Perlman et al. (2001), Sleptchenko et al. (2002), Sleptchenko et al. (2005), Perlman and Kaspi (2007), and Adan et al. (2009) study stochastic multi-echelon inventory systems with several repair modes. All of these models assume that failures occur according to a Poisson process with a constant rate. However, in many practical situations there is no convincing argument to justify the latter assumption, especially if the repair process is not steady. On the other hand, for almost any practical situation, the inventory manager can give an accurate deterministic estimate of the lower and upper bounds on the inventory level in any period over the planning horizon. We study such type of deterministic bounded (i.e., interval-valued) demand in this paper.

To the best of our knowledge, not much has been done to date on modeling the planning and scheduling of spare-part inventories with deterministic demand. Most of the related studies focus on the very special case involving a single supplier and a fixed-demand inventory (e.g., Prager, 1956; Federgruen, 1993; Gass, 2003; Abdul-Jalbar et al., 2006; Federgruen et al., 2007; Perlman and Levner, 2010). The models are generally formulated as a cost minimization problem, with a cost function comprising the holding cost, ordering/setup cost, and either an explicit penalty cost or a specified service level constraint. The models in the literature can be divided into supply with or without outsourcing, and backorders versus lost sales, while the models with backorders can be further divided into cost versus

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service level requirements. Such models have been applied to handle production planning and scheduling in practice. They often employ lot-sizing algorithms, ranging from simple heuristics to complicated dynamic programming, network flow formulation, and integer programming, as solution methods. A multi-echelon inventory model implies the existence of a hierarchy of stocking locations and interaction among their inventories.

In this paper we study a repair shop operating under the assumption that supply and demand in the forthcoming periods are predictable and deterministic. This allows us to determine the optimal safety stock at the depot and the optimal levels of orders of spare parts from the external supplier. We suggest taking a network approach to model the multi-echelon, multi-supplier problem. The solution method extends the early deterministic spare-part management models suggested by Zangwill (1969), Golany et al. (2001), Gass (2003), Liu et al. (2005), Liu and Zhang (2006), and Ahuja and Hochbaum (2008). Their approaches are also based on using network models, but their models are developed only for two-echelon inventory systems with fixed numerical data and cannot treat interval-valued supply and demand. Our network approach generalizes an early supply model known in the literature as the *Caterer Problem* studied by Prager (1956), Ford and Fulkerson (1962), and Gass (2003).

We consider a repairable-item inventory system in which the number of daily failures at each base may change from day to day. We call these entering (failed) items *supplies*. Similarly, we assume that the demand for good items, which are required at each base every day, also change from day to day. Both the daily supply $R_t(k)$ and daily demand $F_t(k)$ can be predicted, but up to a limited degree of precision that is known in advance. Specifically, we assume $R_t(k)=R_t^0(k)\pm\Delta R_t^0(k)$ and $F_t(k)=F_t^0(k)\pm\Delta F_t^0(k)$, where $R_t^0(k)$, $F_t^0(k)$, $\Delta R_t^0(k)$, and $\Delta F_t^0(k)$ are known entities. It follows that, for each base $k$, there are given prescribed intervals, called the *supply window*: $\mathrm{R}_t(k)=[R_t^l(k), R_t^u(k)]$ and the *demand window*: $\mathrm{F}_t(k)=[F_t^l(k), F_t^u(k)]$, in which the daily supply and daily demand lie, respectively. Here the lower bound $F_t^l(k)$ is a forecast of the minimum demand for good items that should be available at base $k$ on day $t$. On the other hand, the upper bound $F_t^u(k)$ is the maximum demand allowed at base $k$ on day $t$. If the stock required to fulfill this demand is insufficient on a certain day, the unfulfilled demand is *backordered*. The backorder is fulfilled later when new items arrive from the external supplier or when the repair shop fixes the failed items. The expected number of backorders, as well as the backorder cost and the number of backorder days, is an important measure of the effectiveness of inventory management as discussed by Liberopoulos et al. (2010). If the number of entering items plus the available stock at the depot are insufficient to fulfill the total demand, then the items are outsourced from the external supplier, which incurs a higher cost.

The objective of our study is to develop a model to determine the optimal number of spare parts to purchase from the external supplier and the optimal number of failed items to be repaired daily (and stored if necessary) in order to meet the required demand at the minimum operating cost.

Our model has three main features. First, we assume supply and demand are predictable up to a limited degree of precision. For this situation we formulate a network-flow model with lower and upper flow bounds to optimize the circulation of spare parts and decide the repair option. Second, we explicitly incorporate transportation time/cost and repair time/cost into the repair process. Third, we implicitly introduce multiple suppliers, as well as backorder time and penalty into the network model.

The rest of the paper is organized as follows: In the next section we present a mathematical model of the problem under study and show that it is equivalent to a minimum-cost network flow problem on a special network with lower and upper bounds. In Section 3 we study the structural properties of the optimal solution and decompose the initial problem into two independent sub-problems. In Sections 4 and 5 we present a solution algorithm and provide several numerical examples to illustrate the solution procedure, respectively. We conclude the paper and suggest topics for future research in Section 6.

## 2. The model

In this section we describe the spare-part supply system under study, introduce the parameters and variables, and formulate the objective function and constraints.

### 2.1. System description

We consider the scenario in which there are several operational sites (bases) served by a repair shop (depot) with a storage facility, where spare parts are kept. The stock at the depot can also be filled from an external supplier (see Fig. 1 borrowed from Perlman and Levner, 2010).

When there is demand for the spare part at a base, if there is sufficient stock in the depot, replacement items are sent through

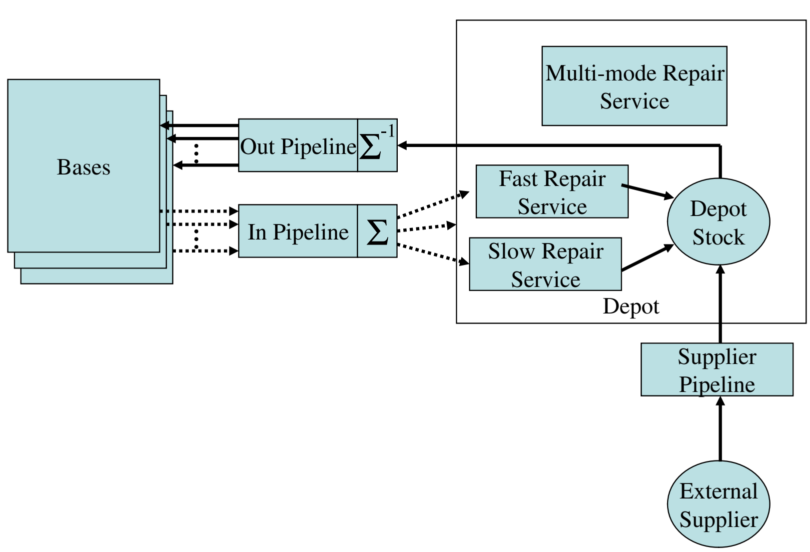


**Fig. 1.** Flow of parts in a multi-echelon system (Perlman and Levner, 2010).