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A fuzzy random continuous review inventory system

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ABSTRACT

A fuzzy random continuous review system has been presented in this paper with the annual customer demand assumed to be a uniformly distributed continuous fuzzy random variable. Besides the reorder point and the production lot size, the setup cost and the 'out of control' probability for a production process have been assumed to be control parameters in the model. Investments to reduce the setup cost and improve the process quality have been incorporated into the total cost in this regard. A methodology has been proposed to minimize this cost and it has been illustrated by way of a numerical example.

1. Introduction

Over the years, in the study of classical production/inventory problems, the items produced have been assumed to be of the same optimal quality. However, in real production environment, it is often observed that the product quality is not always perfect as it usually depends on the state of the production process. The setup cost is also assumed to be constant in the analysis of production/inventory models. However in reality it is not so. This is because the setup cost can be controlled and reduced through various means such as worker training, procedural changes and advanced equipment acquisition. Besides, reduced setup cost also leads to lower lot sizes, which in turn provide many advantages such as lower inventory, shorter lead-time, improved quality and many others as outlined by Hong and Hayya (1993). Also in the last two decades or so, it has been shown that reducing setup cost and improving quality also lead to the successful application of the Japanese manufacturing philosophy of just-in-time (JIT) production. This is due to the fact that from the viewpoint of inventory management, the goal of JIT is to produce smaller lot sizes of good quality products. So when sufficient investments are made to reduce the setup cost and improve the quality of the products, the goal of JIT is naturally realized. Therefore it is important to consider these issues for the development of realistic production/inventory models.

Porteus (1986) and Rosenblatt and Lee (1986) were among the first researchers to have taken up the problem of quality improvement and its relation to lot size. Later this issue was further analyzed by researchers. For instance, Keller and Noori (1988) studied the impact of investing in quality improvement in lot size models. Later researchers like Hwang et al. (1993), Hong et al. (1993), Moon (1994), Hong and Hayya (1995) and Voros (1999) also made significant contributions in this regard. The problem of setup cost reduction was also first put forward by Porteus (1985). His work inspired many researchers like Lee et al. (1997), Kim and Hong (1999), Hou and Lin (2004), Hou (2007), Kulkarni (2008), etc. to take up this issue in various kinds of inventory models. Recently, Ouyang et al. (2002) analyzed the combined effects of lead-time reduction, setup cost reduction and quality improvement in the lot size reorder point model. Affisco et al. (2002) studied the simultaneous effect of setup cost reduction and quality improvement in the joint economic lot size model. Liu and Cetinkaya (2007) made some modifications to the work done by Affisco et al. (2002). However all the above mentioned works are in the probabilistic framework. In these models, the uncertainty inherent in reality is accounted for but the imprecision arising due to vague information is not taken into account. Now to capture this imprecision, fuzzy set theory has come to be widely applied in the study of various inventory models in general and the continuous review system in particular (Hseih, 2002; Kao and Hsu, 2002; Tutuncu et al., 2008; Kumaran and Vijayan, 2008). However as mentioned earlier, the probabilistic models do not consider any imprecision in the model environment while the fuzzy models do not consider any random fluctuations that are so intrinsic to the inventory parameters. Thus to account for both these types of uncertainties, inventory models have now come to be developed in the fuzzy random framework (Dutta et al., 2005; Dey and Chakraborty, 2008, 2009). In particular, Chang et al. (2006) and Dutta et al. (2007) have developed the continuous review systems in the fuzzy random framework. However to the best of the authors' knowledge, the issues of process quality improvement and setup cost reduction in the review systems have not been analyzed in the fuzzy random framework till date. Also, the fuzzy random variable customer demand has been assumed to be of the form $(\tilde{D}_i,p_i),\ i=1,2,\dots,n$ by Dutta et al. (2007), i.e., the fuzzy random variable is of the discrete type. However, in most real life inventory situations, especially if there are sufficient or abundant data, then assuming the demand to

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be of the aforementioned form makes the related mathematical calculations very cumbersome and time consuming. Further, if the statistical data itself possesses fuzziness in terms of unreliable or inaccurate data (lack of proper documentation), lack of precise information (linguistic expression), non-constant reproduction conditions (variability of inventory situation), etc., then it is computationally much easier and intuitively pleasing as well to assume the fuzzy random demand to follow some known continuous distribution. Thus, with this point of view, the customer demand has been assumed to be a continuous fuzzy random variable of the form $\tilde{D}(\omega)=(D(\omega)-\Delta_1,\ D(\omega), D(\omega)+\Delta_2)$ (Liu and Liu, 2003), where $\omega\in\Omega$ and (Ω, B, P) is a probability space. Here Δ_1 and Δ_2 are the left and right spreads, respectively, with $0<\Delta_1<D(\omega)$ and $\Delta_2>0$ for all $\omega\in\Omega$, where $D(\omega)$ follows some continuous distribution. The values of Δ_1 and Δ_2 are set by the decision maker as per his judgment of the situation. This allows him to include his experience into the model building exercise. It also lends additional flexibility in terms of application of the model to specific inventory situations.

Thus, in this paper a continuous review inventory model has been developed under the fuzzy random framework assuming the annual customer demand to be a continuous fuzzy random variable following uniform distribution. Besides the reorder point and the order quantity, the setup cost and the 'out of control' probability for a production process have been assumed to be the control parameters. A methodology has been developed such that the total cost incurred is a minimum and the optimal values of the control parameters are determined in the process.

The subsequent sections have been organized as follows: Section 2 outlines the preliminary concepts that have been used for model building purposes. In Section 3, the model has been developed methodology formulated. A numerical example illustrates the methodology in Section 4 and finally some concluding remarks have been made in Section 5.

2. Preliminary concepts

2.1. Triangular fuzzy numbers

A normalized triangular fuzzy number $\tilde{A}=(\underline{a},a,\overline{a})$, where $\underline{a},a,\overline{a}$ are real numbers, is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

(i) $\mu_{\tilde{A}}(x)$ is continuous mapping from $\mathbb{R}$ to the closed interval [0,1];
(ii) $\mu_{\tilde{A}}(x)=L(x)=x-\underline{a}/a-\underline{a},\quad \underline{a}\le x\le a$ is strictly increasing on $[\underline{a},a]$;
(iii) $\mu_{\tilde{A}}(x)=1,\quad x=a$;
(iv) $\mu_{\tilde{A}}(x)=R(x)=\overline{a}-x/\overline{a}-a,\quad a\le x\le \overline{a}$ is strictly decreasing on $[a,\overline{a}]$ and
(v) $\mu_{\tilde{A}}(x)=0$, elsewhere.

where $\underline{a}$, a, $\overline{a}$ are the real numbers and $L(x)$ and $R(x)$ are the left and right shape functions, respectively. A normal triangular fuzzy number $\tilde{A}$ may also be denoted by its α-cut as $A=[A_\alpha^-,A_\alpha^+]$, where $\alpha\in[0,1]$. The α-cut representation and the triangular notation are then connected as follows:

$$A_\alpha^-=\underline{a}+\alpha(a-\underline{a})\ \text{and}\ A_\alpha^+=\overline{a}-\alpha(\overline{a}-a) \tag{1}$$

Without any loss of generality, all fuzzy quantities have been assumed to be triangular normal fuzzy numbers throughout this paper.

2.2. Possibilistic mean value of a fuzzy number

For a given fuzzy number $\tilde{A}$, the interval-valued possibilistic mean is defined by Carlsson and Fuller (2001) as $M(\tilde{A})=[M_*(\tilde{A}),\quad M^*(\tilde{A})]$, where $M_*(\tilde{A})$ and $M^*(\tilde{A})$ are the lower and upper possibilistic mean values of $\tilde{A}$ and are, respectively, defined by

$$M_*(\tilde{A})=\frac{\int_0^1 \alpha A_\alpha^- d\alpha}{\int_0^1 \alpha d\alpha}\ \text{and}\ M^*(\tilde{A})=\frac{\int_0^1 \alpha A_\alpha^+ d\alpha}{\int_0^1 \alpha d\alpha}$$

The possibilistic mean value of $\tilde{A}$ is then defined as

$$\overline{M}(\tilde{A})=(M_*(\tilde{A})+M^*(\tilde{A}))/2$$

In other words, it can be written as

$$\overline{M}(\tilde{A})=\int_0^1 \alpha(A_\alpha^-+A_\alpha^+)d\alpha \tag{2}$$

Therefore for a fuzzy number $\tilde{A}=(\underline{a},a,\overline{a})$, the possibilistic mean takes the form:

$$\overline{M}(\tilde{A})=\int_0^1 \alpha(A_\alpha^-+A_\alpha^+)d\alpha=\int_0^1 \alpha\{\underline{a}+\alpha(a-\underline{a})+\overline{a}-\alpha(\overline{a}-a)\}d\alpha=\frac{\underline{a}+\overline{a}}{6}+\frac{2a}{3} \tag{3}$$

Now if $\tilde{A}$ and $\tilde{B}$ were the two fuzzy numbers where $A_\alpha=[A_\alpha^-,A_\alpha^+]$ and $B_\alpha=[B_\alpha^-,B_\alpha^+]$, $\alpha\in[0,1]$, then, for ranking fuzzy numbers, we have $\tilde{A}\le\tilde{B}\Leftrightarrow\overline{M}(\tilde{A})\le\overline{M}(\tilde{B})$.

2.3. Fuzzy random variable and its expectation

Kwakernaak (1978) first introduced fuzzy random variables. Puri and Ralescu (1986) also discussed this concept in later years. Here the definition given by Kwakernaak (1978) and further explained by Gil et al. (2006) has been considered.

Let us consider the p-dimensional Euclidean space $\mathbb{R}^p$. $F(\mathbb{R}^p)$denotes the class of upper semi-continuous function in $[0,1]^{\mathbb{R}^p}$ with compact closure of the support. Then, for the one-dimensional case, $F_C(\mathbb{R})$ is the sub-class of convex sets of $F(\mathbb{R})$. Given a probability space (Ω, A, P), a mapping $\chi:\Omega\to F_c(\mathbb{R})$ is said to be a fuzzy random variable if for all $\alpha\in[0,1]$, the two real-valued mappings inf $\chi_\alpha:\Omega\to(\mathbb{R})$ and sup $\chi_\alpha:\Omega\to(\mathbb{R})$ (defined so that for all $\omega\in\Omega$ we have $\chi_\alpha(\omega)=[\inf(\chi(\omega))_\alpha, \sup(\chi(\omega))_\alpha]$) are the real-valued random variables.

A fuzzy random variable may also be defined as $\tilde{D}(\omega)=(D(\omega)-\Delta_1,D(\omega),D(\omega)+\Delta_2)$ (Liu and Liu, 2003), where $\omega\in\Omega$ and (Ω, B, P) is a probability space. Here Δ_1 and Δ_2 are the left and right spreads, respectively, with $0<\Delta_1<D(\omega)$ and $\Delta_2>0$ for all $\omega\in\Omega$, where $D(\omega)$ follows some continuous distribution.

The fuzzy expectation of a fuzzy random variable is a unique fuzzy number. It is defined as

$$E\tilde{X}(\omega)=\int_\Omega \tilde{X}(\omega)dP=\left\{\left[\int_\Omega X_\alpha^-(\omega)dP,\int_\Omega X_\alpha^+(\omega)dP\right]/0\le\alpha\le1\right\}$$

where the fuzzy random variable is $[X]_\alpha=[X_\alpha^-,X_\alpha^+],\quad \alpha\in[0,1]$.

The α-cut of the fuzzy expectation is given by

$$[E\tilde{X}(\omega)]_\alpha=E[X(\omega)]_\alpha=\{[E(X_\alpha^-(\omega)),E(X_\alpha^+(\omega))],\alpha\in[0,1]\}$$

3. Methodology

3.1. Model and assumptions

The efficient control of inventory is essential for the smooth running of any business. For the management of single item inventories, the continuous review inventory system is of primary importance. There are a number of operating policies in the continuous review system, as discussed by Hadley-Whitin (1963). One of the most widely used doctrines is the (Q,r) review system. In this doctrine, when the inventory level reaches a certain critical