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The cost of using stationary inventory policies when demand is non-stationary

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ABSTRACT

Non-stationary stochastic demands are very common in industrial settings with seasonal patterns, trends, business cycles, and limited-life items. In such cases, the optimal inventory control policies are also non-stationary. However, due to high computational complexity, non-stationary inventory policies are not usually preferred in real-life applications. In this paper, we investigate the cost of using a stationary policy as an approximation to the optimal non-stationary one. Our numerical study points to two important results: (i) Using stationary policies can be very expensive depending on the magnitude of demand variability. (ii) Stationary policies may be efficient approximations to optimal non-stationary policies when demand information contains high uncertainty, setup costs are high and penalty costs are low.

1. Introduction

Stochastic inventory control systems have been studied extensively under various assumptions on demand. Nevertheless, the literature reflects a clear dichotomy between inventory models with stationary and non-stationary demands. The former assumes a steady demand process, whereas the latter assumes a demand process that varies in time. Strictly speaking, most practical demand patterns are non-stationary [1]. Furthermore, as product life cycles are becoming shorter, demand that evolves over the life of the product never follows stationary patterns [2]. For instance, electronic products, which have relatively short life cycle, generally follow non-stationary demand patterns (see e.g. [2,3]). Moreover, many authors have reported that providers of components and subassemblies often face unstable customer orders (see e.g. [4,5]).

One major theme in the continuing development of inventory theory is the incorporation of more realistic demand assumptions into inventory models. Consequently, one would expect increasing number of studies concerned with non-stationary inventory models. However, the literature on non-stationary demand is rather limited, whereas it is vast for stationary demand. A topic search (title, abstract and keywords) on the ISI Web of Knowledge, since the year 2000, using the terms *stationary* and *inventory* gives 221 published papers, whilst this figure is only 29 for the terms *non-stationary* and *inventory*. It is obvious that, there is also a large number of papers assuming stationary demand without using the term *stationary*. This disparity is mainly due to the ill structure of non-stationary problems from a theoretical point of view and the complexity inherent in non-stationary models from a computational point of view. Silver et al. [6] point out that non-stationary demand is too complicated for routine use in practice. Furthermore, as Kurawarwala and Matsuo [7] stated, the unique characteristics of non-stationary demand preclude the use of traditional forecasting methods not designed for this environment and raise a need for tailor-made forecasting methods. Consequently, stationary policies have always been preferred to non-stationary policies in many real-life applications for the sake of their relative simplicity even if the underlying actual demand is non-stationary.

In spite of all the above mentioned issues related to non-stationary inventory policies, when demand is non-stationary, a stationary policy is an approximation to the optimal non-stationary one, and hence, is sub-optimal with respect to total expected cost. This research investigates the magnitude of this sub-optimality under various settings. To the best of our knowledge, no work has been done that can be used as a guideline to compute the cost of using stationary policies when demand is non-stationary. We establish our analysis by using the (*s*,*S*) inventory control policy. The (*s*,*S*) policy is proven to be optimal both in stationary and non-stationary demand cases, and therefore, constitutes an inherent frame of reference. Our contribution is two-fold. First, we show that using stationary policies can be very expensive depending on the extent of demand variability as well as other factors. Second, we provide some insight on cases where stationary models provide good approximations to non-stationary models.

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In the remainder of this section, we concisely review related literature. In Section 2, we give the key assumptions of the inventory problem considered, and present algorithms used to compute the stationary and the non-stationary (*s*,*S*) policies. In Section 3, we present the experimental design and computational results. Finally, in Section 4, we draw general conclusions and provide some managerial insights.

Most of the research in inventory literature assumes either a stationary or a non-stationary demand, and develop models and policies accordingly. Therefore, it is difficult to refer to any research addressing the cost performance of stationary policies when demand is non-stationary. However, we believe that it is necessary to briefly discuss the key literature in order to ease the exposition of the remaining sections.

One of the most exciting developments in the inventory theory is Scarf's [8] proof of the optimality of (*s*,*S*) policies. (*s*,*S*) policies are characterized by two critical numbers s_n and S_n for each period *n*, such that, the inventory position is replenished up to a target level S_n whenever the inventory position at the beginning of the period is lower than (or equal to) a re-order level s_n. Scarf [8] showed the optimal value function satisfies a condition, which he called *K*-convexity, and provided a procedure for establishing the optimal policy parameters via a recursive function. Scarf's formulation required extensive computational power beyond the limitations of its time. As a matter of fact, there was no known way of computing policy parameters at that time [9]. Following Scarf [8], Iglehart [10] demonstrated the optimality of (*s*,*S*) policies in infinite horizon inventory problems with stationary demand. He showed that optimal policy parameters converge to two limit values *s* and *S* in this case. Iglehart's work has been followed by a large number of researchers (see e.g. [11–17]) aiming at efficiently computing optimal policy parameters using the stationary analysis approach. However, not much work has been done for computing non-stationary (*s*,*S*) policies. A few authors addressed the inventory problem with non-stationary demands. Some of these work focused on alternative inventory control policies (see e.g. [18–21]), whereas some others proposed heuristics for computing near-optimal (*s*,*S*) parameters (see e.g. [22]). In this paper, we consider the inventory problem addressed in Scarf [8] and investigate the cost efficiency of stationary and non-stationary inventory policies.

2. Problem definition and solution procedures

In this section, we provide the grounds to investigate the cost performance of stationary policies under non-stationary demand. We establish our analysis by evaluating the best possible stationary policy, i.e. the policy providing the minimal cost for the given non-stationary demand, against the best non-stationary policy. We use the (*s*,*S*) policy as a frame of reference since it is proven to be optimal both in stationary and non-stationary demand cases.

Throughout the paper it is assumed that the planning horizon consists of *N* periods. The demand, d_n in period *n*, is a random variable with known probability density function, $g_n(d_n)$, and occurs instantaneously at the beginning of the period. The demand rate may vary from period to period. Demands in different time periods are independent. A fixed holding cost *h* is incurred on any unit carried in inventory from one period to the next. Demands occurring when the system is out of stock are backordered, and satisfied immediately when the next replenishment order arrives. A fixed shortage cost *p* is incurred for each unit of demand backordered. A fixed procurement (ordering or set-up) cost *K* is incurred each time a replenishment order is placed. For convenience, without loss of generality, the initial inventory level and the unit procurement cost are set to zero. It is also assumed that there is no replenishment lead-time. However, a brief discussion of positive lead-time is presented in the conclusion section.

2.1. The optimal non-stationary (s,S) policy

Scarf [8] developed the concept of *K*-convexity and proved that under the aforementioned assumptions the optimal inventory policy follows an (*s*,*S*) rule. He provided a dynamic programming formulation to compute the optimal (*s*,*S*) levels for each period. Obviously, (*s*,*S*) levels are not constant for different periods in non-stationary problems. Thus, parameters of a non-stationary policy can be represented as (s_n,S_n) for period *n*. The dynamic program proposed by Scarf is given below

$$C_n(x)=\min\{L_n(x)+\mathrm{E}\{C_{n+1}(x-d_n)\},K+L_n(S_n)+\mathrm{E}\{C_{n+1}(S_n-d_n)\}\} \quad (1)$$

The state variable *x* is the inventory position at the beginning of the time period. $C_n(x)$ denotes the expected cost of following the optimal policy from period *n* onwards, $L_n(x)$ represents the expected period cost function if the opening inventory position is *x*. It is extremely difficult to find optimal (s_n,S_n) levels when the state space of *x* is continuous. Relatively recently, Bollapragada and Morton [22] compute optimal (s_n,S_n) levels by restricting the state space of *x* to integer values. In this paper, we employ their approach to determine the optimal non-stationary (*s*,*S*) policy.

2.2. Best representative stationary (s,S) policy

One may think of two possible approaches to obtain the best stationary policy for a non-stationary demand pattern. The first approach is to find a stationary demand distribution which best fits the original demand for the given inventory system. Once the best stationary demand distribution is determined, the corresponding stationary policy can be computed using the algorithm proposed by Zheng and Federgruen [17]. The second approach is to find a stationary policy which provides the minimum cost for the actual non-stationary demand. To the best of our knowledge, there is no published work in the literature on either one of these approaches. Therefore we settle to employ an exhaustive search procedure. The aforementioned approaches differ in their search spaces, such that, the former approach requires a search on various demand patterns, whereas the latter requires a search on various policy parameters. Since characterizing the search space of the first approach is rather difficult compared to the second one, we employ the second approach, and compute the best stationary policy through a two-dimensional search procedure on integer valued (*s*,*S*) couples with $s\leq S$. The expected cost of each (*s*,*S*) pair is examined by means of the recursive formulation given in Eq. (1) without considering minimization.

3. Numerical study

The experiment design, results and their interpretation are crucial to understand the application of stationary policies in non-stationary demand environments. In the next subsections these will be given in detail.

3.1. Experiment design

In the experiment design phase we concentrate on: (i) end-of-horizon effects, and (ii) demand and cost parameters.