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An investigation of setup instability in non-stationary stochastic inventory systems

Onur A. Kilic [a,b,*], S. Armagan Tarim [b,1]

[a] Department of Operations, University of Groningen, P.O. Box 800, 9700 AV, Groningen, The Netherlands
[b] Department of Management, Hacettepe University, 06800 Beytepe, Ankara, Turkey

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ABSTRACT

In stochastic inventory systems unfolding uncertainties in demand lead to the revision of earlier replenishment plans which in turn results in an instability or so-called system nervousness. In this paper, we provide the grounds for measuring system nervousness in non-stationary demand environments, and gauge the stability and the cost performances of (R,S) and (s,S) inventory policies. Our results reveal that, both the stability and the cost performance of inventory policies are affected by the demand pattern as well as the cost parameters, and the (R,S) policy has the potential to replace the cost-optimal (s,S) policy for systems with limited flexibility.

1. Introduction

In inventory planning systems, inventory replenishment plans, i.e. the timing and the size of replenishments, are often revised in response to realized demand. In practice, when this is the case, the replenishment plan is regenerated for the rest of the planning horizon leading to a planning instability or so-called *system nervousness* (Vollmann et al., 1988).

There is a large variety of inventory policies applied in inventory management practices (see e.g. Silver et al., 1998). These policies are extensively investigated in terms of their cost performance. However, in systems with low degrees of flexibility, the cost of implementing revisions in replenishment decisions may overcome the advantage of using the cost-efficient technique. In this context inventory control rules show different levels of instability. Thus system nervousness, as a performance criterion, can be of high importance in assessing inventory control rules. Omitting the planning instability can turn out to be a serious problem because it gives rise to a considerable amount of alteration efforts (Heisig, 2001).

The (s,S) policy has been shown to be cost-optimal under very relaxed assumptions in both stationary and non-stationary demand cases (see Scarf, 1960; Iglehart, 1963). Heosog (1998, 2001), and de Kok and Inderfurth (1997) have questioned the performance of the (s,S) policy with respect to the nervousness criterion in the stationary demand case. Their research reveals the trade-off between cost effectiveness and nervousness and shows that the (s,S) policy exhibits the worst stability performance among a number of policies considered. Different strategies for dealing with the problem of nervousness are examined by Blackburn et al. (1986). They suggest an effective strategy based on freezing certain orders so they cannot be changed. In this regard, the (R,S) policy, in which the timing of future orders are fixed, provides a means of dampening the setup instability. Silver et al. (1998) points out that the (R,S) policy, which provides a rhythmic rather than a random replenishment pattern, is usually appealing from a practitioners point of view.

One major difficulty in the continuing development of inventory theory is to incorporate more realistic assumptions about demand into inventory models. In many stable environments it is an adequate approximation to treat period demands as identically distributed random variables. However, demand patterns are often heavily seasonal or possess significant trends especially in industrial settings with business cycles. Furthermore, as product life cycles get shorter, the randomness and the unpredictability of demand increases. The essence of such situations can only be captured by means of finite horizon non-stationary inventory models.

Literature provides guidelines about the stability performance of inventory policies in stationary systems. However, those may not be directly generalized to non-stationary systems. In stationary systems policy parameters are also stationary, and therefore, the measure of stability of the whole system can be determined by means of observing two arbitrary consecutive planning cycles. However, in non-stationary systems policy parameters are determined in connection with each and every period through the planning horizon, and consequently, stability

* Corresponding author at: Department of Operations, University of Groningen, P.O. Box 800, 9700 AV, Groningen, The Netherlands. Tel.: +31 50 363 8231; fax: +31 50 363 2032.
E-mail address: o.a.kilic@rug.nl (O.A. Kilic).
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is a function of the demand pattern (Blackburn et al., 1986). To the best of our knowledge, no work has been done on the measures of nervousness in non-stationary systems. In this paper, we aim to fill in this gap by investigating the system nervousness under non-stationary stochastic demand. Our contribution is two-fold. First, we propose a method for measuring system nervousness in non-stationary demand environments. Secondly, we gauge the setup stability of (R,S) and (s,S) inventory policies and demonstrate that the (R,S) policy has the potential to replace the cost-optimal (s,S) policy, especially for systems characterized by a low degree of flexibility to setup changes.

The remainder of this paper is organized as follows. Section 2 investigates the related literature and positions the current work. Section 3 defines the inventory system addressed. Section 4 provides the grounds for computing setup instability in non-stationary environments and proposes a method for the computation thereof. Section 5 introduces the models computing (s,S) and (R,S) policies. Section 6 presents the computational experiments. Section 7 concludes and sketches some likely extensions of the study.

2. Background and positioning

The definition of nervousness plays a major role in the investigation of the performance of inventory control rules with respect to stability. Previous research specify two classes of general nervousness types: short/long term and setup/quantity oriented nervousness. The former involves rating planning instability between the first two periods for the short term and between all consecutive periods for the long term. The latter distinguishes between considering adjustments on pure replenishment actions (cancelation of a scheduled order or placement of an unscheduled order) and adjustments on the replenishment quantities. It is noted by Inderfurth (1994) and Heisig (2001) that setup oriented system nervousness is considered as the most serious in practice. In this paper we address long-term setup-oriented nervousness which we will refer to as *setup instability*.

It is often difficult to express nervousness in terms of cost. For this reason, rather than integrating nervousness into pure cost-based inventory models, like for instance in Kropp et al. (1983) and Kropp and Carlson (1984), we define stability as an independent attribute of an inventory control system and refer to the measures used in Jensen (1996) and Heisig (2001). Early studies in nervousness involves a wide set of simulation studies, where the impact of different planning parameters on system nervousness are investigated (see e.g. Blackburn et al., 1986, 1987; Sridharan et al., 1988; Minfie and Davis, 1990; Kadipasalioglu and Sridharan, 1997). A systematic development of nervousness measures in stochastic environments is given in Inderfurth (1994), de Kok and Inderfurth (1997), Heisig (1998) and Heisig (2001). Analytical results are presented in Inderfurth (1994) where the performances of (s,S) and (s,nQ) policies with respect to short-term setup-oriented nervousness is analyzed. In de Kok and Inderfurth (1997), the short-term setup-oriented as well as the short-term quantity-oriented nervousness are examined for (s,S), (s,nQ) and (R,S) policies. In Heisig (1998) and Heisig (2001) long-term setup-oriented nervousness performances of (s,S) and (s,nQ) policies are analyzed. Above mentioned studies define instability as the ratio of expected deviations over the maximum deviations that can take place in the worst case. These studies show that there exists a trade-off between cost effectiveness and stability performance and the (s,S) policy which is optimal in terms of cost performance exhibits the worst stability performance among all other policies considered.

Previous studies investigate nervousness in the *rolling horizon* framework. In this framework, although setups and associated quantities are computed over the entire planning horizon, only the first period decision is implemented, and then the schedule is rolled forward to the next period with new demand appended to the horizon. Within this approach, there are two sources of nervousness: demand uncertainty and rolling horizon planning (Kadipasalioglu and Sridharan, 1997). Demand uncertainty implies that actual demand may differ from the forecast, and therefore, leads to a revision of setups as necessary. Rolling horizon planning, however, may cause planned orders to change because of the new information obtained about future demands. In this study we adapt a *re-planning* approach rather than a rolling horizon framework. The main difference between these two is that in a re-planning approach new periods are not appended to the fixed length planning horizon and replenishment plans are generated only for the remaining periods. One positive side effect of this approach is that the inventory system is no longer exposed to the instability due to the rolling horizon planning. Hence it gives us the opportunity to investigate the effect of demand uncertainty on stability performance in isolation.

3. System configuration

We address a multi-period stochastic inventory problem which is characterized by a finite planning horizon comprising N periods. The demand, d_t in period t is considered as a discrete random variable with known probability mass function, $g_t(d_t)$, and occur instantaneously at the beginning of each period. The demand distribution may vary from period to period. Demands in different time periods are assumed independent. A fixed holding cost h is incurred on any unit carried in inventory over from one period to the next. Demands occurring when the system is out of stock are backordered, and satisfied immediately when the next replenishment order arrives. A fixed shortage cost b is incurred for each unit of demand backordered. A fixed setup cost K is incurred each time a replenishment order is placed, whatever the size of the order. For the purposes of the study, the direct item cost is assumed to be zero and the delivery lead-time is not incorporated.

We assume that replenishment plans are made and updated in response to realized demands throughout the planning horizon. The replenishment plans as well as the timing of re-planning actions depend on the characteristics of the employed inventory policy. The sequence of events at an arbitrary re-planning epoch is given as follows. Initially, the state of the inventory system (i.e. the current period and the closing inventory level at the previous period) is observed and the optimal replenishment plan for the rest of the horizon is computed according to the inventory policy used. The optimal replenishment plan specifies the replenishment periods through the rest of the planning horizon and the timing of the next re-planning epoch. Then, following the re-planning concept, the part of the replenishment plan concerning the periods until the next re-planning epoch is put into action. That is, an immediate replenishment order is issued and received if necessary and the consequent inventory is used to serve the demand until the next re-planning epoch where the inventory plan is revised again. Within this context, no-more-optimal inventory plans are replaced by optimal ones in successive re-planning actions.

For any given inventory policy, the above explained inventory system can be expressed as a stochastic process defined over the state space Ξ characterizing possible states where re-planning actions may take place and a probability matrix P characterizing the transition probabilities between those states. Let us consider