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An integrated production–distribution model for a deteriorating inventory item

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ABSTRACT

We develop an integrated production–distribution model for a deteriorating item in a two-echelon supply chain. The supplier's production batch size is restricted to an integer multiple of the discrete delivery lot quantity to the buyer. Exact cost functions for the supplier, the buyer and the entire supply chain are developed. These lead to the determination of individual optimal policies, as well as the optimal policy for the overall, integrated supply chain. We outline a procedure for determining the optimal supply chain decisions with the objective of minimizing the total system cost. Our approach is illustrated through a numerical example.

1. Introduction

In real life, it is not uncommon for inventory items, such as milk, fruit, blood, pharmaceutical product, vegetables etc, to decay or deteriorate over time. Therefore, it is important to study the behavior of such decaying and deteriorating items, towards the formulation of appropriate inventory control policies that explicitly take such behavior into account.

Ghare and Schrader (1963) were the first authors to consider the effect of decay on inventory items. They used the term "inventory decay" to describe this phenomenon, including direct spoilage, physical depletion and deterioration. They developed a general EOQ type model under constant demand with exponential decay, which could be solved iteratively, but not directly. Covert and Philip (1973) and Tadikamalla (1978) extended Ghare and Schrader's work to Weibull and gamma distribution based deterioration patterns, respectively. All of the above models assumed instantaneous replenishment and did not allow backorders or shortages. Misra (1975) developed EOQ type models with finite production rates, but did not allow backorders. His models include cases of varying and constant deterioration rates. Shah (1977) developed models for both exponential and Weibull deterioration cases, which allowed backorders albeit with instantaneous replenishment. Mak (1982) developed a production lot size inventory model with backorders for exponentially decaying items. Heng et al. (1991) integrated Misra's and Shah's approaches and developed a model for deteriorating items with finite production rate and backorders.

Deterioration inventory model research has received increasing attentions in recent years, extending existing models with a variety of deterioration patterns, demand functions and backordering policies. In most cases, deterioration is assumed to be a constant fraction of total on-hand inventory. The Weibull distribution has been used to model item decay (Chakrabarty et al., 1998), and some attention has been focused on deteriorating items with expiration dates (Hsu et al., 2006; Lo et al., 2007). Recent work in this area has considered time-varying as well as stock level and price dependent demands. Wee (1993, 1995), Hariga et al. (1997), Bhunia and Maiti (1998), Chung and Tsai (2001), Wang (2002), Balkhi (2004), Teng and Chang (2005), Yang (2005), Shah et al. (2005), Chang et al. (2006), Hou (2006), Wu et al. (2006), Manna and Chaudhuri (2006), Pal et al. (2006) and Lo et al. (2007) provide good examples of inventory models with different assumptions concerning the patterns of demand, deterioration and backordering.

Most existing inventory models for deteriorating items are EOQ type models and consider the different sub-systems in the supply chain independently. Although the notion of cooperation between suppliers and buyers has received more and more attention in the literature, few integrated inventory approaches for deteriorating items have been developed to date. Rau et al. (2003) develop a multi-echelon inventory model for a deteriorating item and derive the optimal joint total cost from an integrated perspective, including the supplier, the producer and the buyer. Yang and Wee (2003) develop a mathematical model for multi-item production lot sizing for deteriorating items.

As mentioned above, the existing literature suggests a variety of inventory models pertaining to deteriorating items. Few of

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them, however, have been developed explicitly for the JIT environment. Yang and Wee (2003) developed an integrated multi-product lot-size inventory model for deteriorating items in JIT environment. Both the lot-splitting of material from the supplier to producer, and the lot-splitting of finished good from producer to multiple buyers are considered. However, the optimal solutions are not given. Furthermore, the differential equations used in their paper are not in point any more when the number of buyers is small, such as 1. This paper considers the single-supplier, single-buyer case, involving a deteriorating inventory item in a JIT environment.

Almost all authors to date use calculus based approaches for solving the inventory models developed for deteriorating items. Such approaches become more complicated in the single-supplier, single-buyer case when cost functions for the supplier and buyer are derived separately. In fact, when a lot is delivered from the supplier to the buyer, at specified "delivery points", the inventory level of the supplier changes suddenly and forms an inflexion on the supplier's inventory function. These inflexions make it difficult to use classical optimization techniques. With the common assumption that the item's deterioration rate is small and its square or higher powers can be neglected, this paper intends to derive the cost functions for the suppler, buyer and the entire supply chain and derive appropriate policies based on an algebraic method.

In short, this paper intends to fill a notable gap in the existing literature concerning inventory models for deteriorating items in JIT environments. It may also be seen as an extension and generalization of the recent work by Kim and Ha (2003).

2. Assumptions and notation

2.1. Assumptions

(a) The operating environment is deterministic;
(b) the suppliers production rate and the demand rate on the buyer are constant;
(c) the inventory item's deterioration is a constant fraction of its on-hand inventory;
(d) the production rate is greater than the demand rate;
(e) the buyer pays transportation and order handling costs;
(f) the cost of the deteriorating item is constant;
(g) shortages are not allowed;
(h) the deterioration rate is sufficiently small, such that its square or higher powers can be ignored.

2.2. Notation

1. For the entire supply chain:
   - N = the number of deliveries per production batch cycle;
   - Q = the production lot size per batch cycle (units);
   - T = total cycle time (in time units);
   - q = delivery lot size (units);
   - d = the item's deterioration rate;
   - C_d = the cost of deterioration per unit ($).
2. For the supplier:
   - P = production rate (units/time unit);
   - C = setup cost for a production batch ($/setup);
   - H_S = inventory holding cost in $/unit/time unit;
   - S_{sup} = area under the supplier's inventory level curve.
3. For the buyer:
   - D = demand rate in units/time unit;
   - A = ordering cost in $/order;
   - H_B = inventory holding cost in $/unit/time unit;
   - F = fixed transportation cost per delivery ($);
   - V = unit variable cost for order handling and receiving ($);
   - S_{buy} = area under the buyer's inventory level curve.

3. Model development

In our proposed scenario, a supplier delivers fixed quantities of a product to a buyer's warehouse at fixed time intervals. Each of these deliveries arrives at the warehouse at the exact time when all items from the previous delivery have just been depleted. The inventory time-plots for the buyer and the supplier are shown in Figs. 1 and 2, respectively.

The total cycle time T can be divided into two components: T_1, the time during which the supplier produces product and T_2, the time during which the supplier does not produce the product. Also we let T_3 be the time between two successive deliveries. The relevant costs to be considered in this study include the following:

(a) setup cost per unit time for the supplier $=C/T$,
(b) holding cost per unit time for the supplier $=H_S S_{sup}/T$,
(c) deterioration cost per unit time for the supplier $=C_d dS_{sup}/T$,
(d) ordering cost per unit for the buyer $=A/T$,
(e) holding cost per unit time for the buyer $=H_B S_{buy}/T$,
(f) transportation and handling cost per unit time for the buyer $=NF+VNq/T$,
(g) deterioration cost per unit time for the buyer $=C_d dS_{buy}/T$.

3.1. The buyer's inventory cost model

During each delivery cycle, denoting x the number of deteriorated units, the delivery lot size is

$$q=x+DT_3$$

The quantity q is divided into two parts: DT_3 is for consumption and x represents the number of deteriorated items. If the deterioration rate is small, we can neglect its square and higher powers. Thus, x can be seen as the deterioration of q units over the interval T_3, i.e.

$$q=DT_3+\frac{dqT_3}{2}$$

Substituting $T_3=T/N$, we obtain

$$T=\frac{2Nq}{2D+dq} \tag{1}$$

Also, considering total deterioration at the buyer's end, we have

$$dS_{buy}=Nq-DT$$

In other words

$$S_{buy}=\frac{Nq-DT}{d} \tag{2}$$

Thus, from Eqs. (1) and (2), the buyer's cost function can be expressed as

$$TC_{buy}(q,N)=\frac{A}{T}+\frac{H_B S_{buy}}{T}+\frac{NF+VNq}{T}+\frac{C_d dS_{buy}}{T}$$
$$=\left(\frac{D}{Nq}+\frac{d}{2N}\right)(A+NF+VNq)+\frac{q}{2}(H_B+C_d d) \tag{3}$$

3.2. The supplier's inventory cost model

If y represents the number of deteriorated units at the supplier's end, we can express

$$y=dS_{\rm sup}$$