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Managing inventory with two suppliers under yield uncertainty and risk aversion

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ABSTRACT

In this paper, we consider a single-product single-period inventory model in which the retailer can source from two suppliers. The primary supplier is cheaper but unreliable in the sense that it generates supply yield uncertainty, whereas the secondary supplier is perfectly reliable but more expensive. The reliable supplier's capacity is fixed and the retailer cannot order more than the quantity reserved in advance. We study the problem in the context of a risk-averse retailer who has to determine the optimal order quantity from the primary supplier and the optimal reserved quantity from the secondary supplier. We develop the model in the perspective of a low risk averse retailer and quantify the risk via an exponential utility function. We show by numerical experiments how the resulting dual sourcing strategies differ from those obtained in the risk-neutral analysis. We also examine the sensitivity of some model-parameters on the optimal decisions.

1. Introduction

Many inventory models have been developed based on the assumption that items are replenished from a single supplier. However, in practice, the sole supplier often fails to meet the retailer's demand due to various reasons. These include insufficient supply of raw materials, production of poor quality products, machine breakdown, workers strike, and so on. Business organizations use a secondary supplier or multiple suppliers today to maintain a desirable service level or to reduce customer service time or even to reduce costs. Dual or multiple sourcing strategy is particularly very useful to retailers for newly launched products which undergo several changes and updates during their early stage of life cycle.

In the supply chain literature, mainly two forms of supply uncertainty have been considered—supply disruption and yield uncertainty. Supply disruption refers to the complete inoperativeness of a portion of the supply chain whereas yield uncertainty refers to a form of supply uncertainty in which the quantity produced or received differs from the quantity ordered by a random amount. Supply disruption models have been studied extensively both for single supplier systems (Parlar and Berkin, 1991; Moinzadeh and Aggarwal, 1997; Arreola-Risa and DeCroix, 1998) and two-supplier systems (Parlar and Perry, 1996; Gurler and Parlar, 1997; Tomlin, 2006). But the majority of yield uncertainty models have been developed for single supplier systems, see Yano and Lee (1995) for a comprehensive review of yield uncertainty literature. Dual sourcing in the context of yield uncertainty has attracted the attention of only a few researchers. Gerchak and Parlar (1990) investigate a second sourcing option in an EOQ (Economic Order Quantity) setting to reduce the effective yield randomness of firm's purchase quantity and deduce conditions under which double sourcing (with distinct yield distributions to two suppliers) is preferable to single sourcing. Parlar and Wang (1993) compare single and double sourcing alternatives in the newsvendor model assuming that actual incoming quantities are a function of random yield. Agrawal and Nahmias (1997) consider a single period supplier selection and order allocation problem with normally distributed supply and show that for two non-identical suppliers, the expected profit function is concave in the number of suppliers. Anupindi and Akella (1993) address the operational issue of quantity allocation between two uncertain suppliers and its effects on the inventory policies of the buyer. Gurnani et al. (2000) simultaneously determine ordering and production decisions for a two component assembly system with random yield from two suppliers, each providing a distinct component. Chopra et al. (2007) develop a single period model integrating two types of supply uncertainty. One supplier is subject to both recurrent and disruption uncertainties and the other one is perfectly reliable. They show that bundling the two uncertainties results in an over-utilization of the unreliable supplier and under-utilization of the reliable supplier.

The above works focus on characterizing the replenishment decisions which optimize the expected cost or profit. That is, the problems are studied from the point of view of risk-neutral decision makers. However, risks due to market fluctuation, high degree of uncertainties in demand and supply, etc. may have a significant impact on cost. For this reason, inventory managers

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sometimes accept a reasonably higher expected cost in order to reduce the variability of cost. So, there is a need to incorporate risk into the managerial decision making. Optimal decisions of a risk-averse retailer in the single-item single-period (newsvendor) problem setting have been extensively studied in the literature, see Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), Keren and Pliskin (2006), Chen et al. (2007), and Chopra et al. (2007); multi-item single period setting (Gotoh and Takano, 2007; Borgonovo and Peccati, 2009); single-item multi-period setting (Bouakiz and Sobel, 1992; Ahmed et al., 2007). The above references utilize mean-variance criterion or expected utility theory to develop models under risk. The exceptions being Ahmed et al. (2007), Gotoh and Takano (2007) and Borgonovo and Peccati (2009) which are based on coherent risk measures.

In this paper, we consider a single-period inventory model for a short-life product which is supplied by two suppliers, one is unreliable and cheaper and other one is reliable but more expensive. The underlying problem scenario is very close to Chopra et al. (2007) in which the delivery quantity from the unreliable supplier is assumed to follow a probability distribution having mean and variance independent of the order quantity. The authors develop the model for a risk-neutral retailer. In this paper, we consider an extended newsvendor model assuming that the mean and variance of random yield are dependent on the order quantity. We develop the model and derive the optimal dual sourcing strategy from the point of view of a risk-averse retailer. We investigate, by numerical experiments, how the resulting dual sourcing strategies differ from those obtained in the standard (risk neutral) mean cost analysis.

2. Notation

The notation used in this paper is as follows:

X	random variable representing the yield
$f(\cdot)$	probability density function for normal distribution with mean μ_s variance σ_s^2
$\phi(\cdot)$	probability density function for standard normal distribution
$\Phi(\cdot)$	cumulative distribution function for standard normal distribution
$\Phi_L(z)=\int_z^\infty (v-z)\phi(v)dv$,	the standard normal loss function where z is the standard normal variate
d	known demand over one period
s	order quantity from Supplier 1
R	reserved quantity from Supplier 2
r	reservation cost per unit
α	probability of delivering each unit by Supplier 1
w_1	purchase cost of each unit from Supplier 1
$w_2(>w_1)$	purchase cost of each unit from Supplier 2
c_o	inventory holding (overage) cost per unit
c_u	shortage penalty (underage) cost per unit

3. Definition of the problem

Suppose that a retail firm is served by two suppliers—Supplier 1 (primary) and Supplier 2 (secondary). Supplier 1 is cheaper but unreliable i.e. there exists a positive probability that the marginal quantity delivered by Supplier 1 is typically less than the quantity ordered. Supplier 2 is more expensive than Supplier 1 but reliable as (s)he always delivers exactly what is ordered. The responsiveness of Supplier 2 allows to place order after observing the response from Supplier 1. However, the firm has to reserve quantities before the order is actually placed to Supplier 2 and it cannot order more than it reserves. Let the firm pay w_1 per unit received from Supplier 1 and w_2 per unit from Supplier 2. Additionally, it pays r per unit to Supplier 2 on the total reserved quantities. We assume that both the suppliers' lead times are zero and $w_2+r>w_1$, that is, it is less expensive to buy a product from the primary (unreliable) supplier. Moreover, (w_2 + r) should not be greater than c_u. Otherwise, the firm will never use the reliable supplier because then the under-stocking cost will be less than the cost of getting the product from the reliable supplier.

Suppose that the firm who faces deterministic demand d orders s units from Supplier 1 and reserves R units in advance from Supplier 2. The quantity it receives from Supplier 1 is a random variable X. If $X<d$, then the firm places an order of size min $\{R,(d-X)\}$ from Supplier 2. That is, if $X<(d-R)$, the firm could order only R units from Supplier 2 and there is a stock-out of $(d-R-X)$. If $(d-R)\le X\le d$, the firm orders $(d-X)$ units from Supplier 2 and there is no under- and over-stock. If, however, $X\ge d$, the firm do not place any order to Supplier 2 and there is an over-stock of $(X-d)$. The problem is to determine the optimal order quantity s and the reserved quantity R set by a risk-averse retailer.

4. Utility function and risk measure

Von Neumann and Morgenstern (1944) developed a model that describes how decision makers choose between uncertain prospects. If a decision maker is able to choose consistently between potential random losses L, then there exists a utility function $u(\cdot)$ to apprise the wealth W such that the decisions (s)he makes are exactly the same as those resulting from comparing the losses L based on the expectation $E[u(W-L)]$. Although it is impossible to determine a decision maker's utility function exactly, we can have some plausible properties of it. For instance, more wealth generally implies a larger utility level, so $u(\cdot)$ should be non-decreasing function. If the decision makers are risk averse, then they have a decreasing marginal utility i.e. $u''(\cdot)\le 0$. In this paper, we model the risk aversion via a specific utility function viz. exponential utility function as suggested by Baker (2006).

Suppose that the utility of a capital of sum ξ is given by

$$u(\xi)=\frac{1-\exp(-\lambda\xi)}{\lambda},$$

where $\lambda(>0)$ can be interpreted as the risk tolerance factor. It is easy to see that u is concave as $u'>0$ and $u''<0$ where prime denotes differentiation with respect to ξ. Moreover, $u\to\xi$ as $\lambda\to 0$. For a cost of amount $\zeta(=-\xi)$, we have the disutility

$$\overline{u}=-u=\frac{\exp(\lambda\zeta)-1}{\lambda}.$$

Let the random variable Y represent the total variable costs of the single-period inventory model under consideration. If Θ denotes the certainty equivalent of Y, then we have

$$\frac{\exp(\lambda\Theta)-1}{\lambda}=\frac{E[\exp(\lambda Y)]-1}{\lambda},$$

which gives

$$\Theta=\frac{\ln(E[\exp(\lambda Y)])}{\lambda}.$$