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# Continuous review inventory model with variable lead time in a fuzzy random environment

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ABSTRACT

Inventory control is an important field in supply chain management, and a great deal of research efforts have been devoted to it over past few decades. In previous researches, there are some assumptions like that the lead time is an incontrollable variable, and all the items replenished are of perfect quality, and so on. However, those assumptions may not be fit for the real environments, and the inventory control problem needs to be considered in a more comprehensive sense. The aims of this paper is to establish the mathematical model and propose an solving approach for the reorder point inventory problems with partial backordered and partial lost sale situation in fuzzy random environment. Specially, the paper investigates the mixture inventory control system in which the lead time demands in different cycles are independent and identically distributed (iid) random variables, and the defective rates of the arrived order lot in different cycles are also iid random variables. Moreover, the backorder rate, ordering cost, shortage penalty cost and marginal profit per unit in different cycles are iid fuzzy variables, respectively. Then based on the fuzzy random renewal reward theory, a mathematical formulation about the expected annual total cost is presented, and some useful properties are analyzed for establishing an efficient solution procedure. In order to calculate the expected value of fuzzy expression and search the optimal values of order quantity, reorder point and lead time, a fuzzy simulation algorithm and an iterative algorithm are designed, respectively. Finally, a numerical example is given to illustrate the procedure of searching the optimal solution.



## 1. Introduction

In today's competitive business environment, many companies recognize the significance of response time as a competitive weapon and have used time as a means of differentiating themselves in the marketplace (Krajewski & Ritzman, 1996). A shorter response time implies faster replenishment, shorter lead time and so on. In inventory system, lead time is the elapsed time between releasing an order and receiving it. In previous literature, lead time is viewed as a prescribed constant or a random variable, which therefore is not subject to control (Silver & Peterson, 1985). In fact, as pointed out in Tersine (1982), lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and setup time. In many practical situations, lead time can be reduced at an added crashing cost; in other words, it is controllable. Moreover, the Japanese successful experience of using Just-In-Time (JIT) production has evidenced that there are substantial advantages and benefits which can be obtained through various efforts of reducing lead time. Recently, much attention has been paid to the lead time reduction by several researchers (e.g., Ben-Daya & Raouf, 1994; Chu, Yang, & Chen, 2005; Liao & Shyu, 1991; Moon & Choi, 1998; Ouyang, Yeh, & Wu, 1996).

In most of the early literature, the quality of the item is neglected, i.e., all the items replenished are of perfect quality. However, it may not always be true. Due to imperfect production process, natural disasters, damage or breakage in transit, etc., the imperfect quality items often appear in replenished orders. If there are imperfect items in orders, which will affect the on-hand inventory level, the number of shortages and the frequency of orders in the inventory system. Thus, it is necessary to take effects of the imperfect items into account in formulating the inventory models. For the continuous reviewed inventory system with imperfect items, Ouyang, Chuang, and Wu (1998), and Wu and Ouyang (2001) considered the problem in random environments and established relevant models, respectively. On the other hand, in real markets, when the inventory system is out of stock, some customers may be willing to wait for filling their demands, while others may fill their demands from another source. That is to say, the demand during the stockout period may be neither completely backordered nor completely lost. Hence, many researchers extended the continuous review inventory models to the cases

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including the partial backordered situation (e.g., Kim & Park, 1985; Montgomery, Bazaraa, & Keswani, 1973; Moon & Choi, 1998; Ouyang et al., 1996). In the researches mentioned above, the fraction of excess demand backordered (or lost) is a fixed constant. However, the backorder rate may be influenced by many factors, such as substitute, customers's preference and waiting patience; in other words, the backorder rate may change slightly due to these potential factors, and it is difficult to measure an exact value for the backorder rate. Therefore, it is appropriate to applying the fuzzy variable to characterize the ambiguous backorder rate. For the cases with fuzzy backorder rate, Ouyang and Chang (2001) presented a model when the lead time demand follows the normal distribution. However, the model in Ouyang and Chang (2001) was not established with deduction but obtained by fuzzifying the backorder rate in the models established preveniently, and the centroid method was applied to defuzzifying the annal cost. In this paper, we try to analyze the inventory system with the exact science, and establish the mathematic model based on the fuzzy random renewal reward theory. Besides, in some real applications, the ordering cost, unit shortage penalty cost and managerial profit per unit investment are hard to express precisely, and more suitably described by linguistic terms such as approximately equal to certain amount subjectively estimated by the expert, and the fuzzy theory provides an appropriate solution approach. In last thirty years, several researchers have developed various types of inventory problems in fuzzy environments (e.g. Dey, Kar, & Maiti, 2005; Hsieh, 2002; Kacprzyk & Staniewski, 1982; Petrović, Petrović, & Vujošević, 1996; Sommer, 1981; Vujošević, Petrović, & Petrović, 1996; Yao & Chiang, 2003 and references therein).

In real inventory systems, the uncertainties such as randomness and fuzzyness are merged with each other. Frequently, we characterize the demand as a random variable and the costs are fuzzy variables in accordance with the practical problems. Several researchers, such as Chang, Yao, and Ouyang (2006), Dutta, Chakraborty, and Roya (2007), Wang, Tang, and Zhao (2007) have considered the problem with the mixture of randomness and fuzziness. Chang et al. (2006) considered the mixture inventory model involving variable lead time with backorders and lost sales. The authors fuzzified the lead time demand to be a fuzzy random variable and the total demand to be the triangular fuzzy number, then presented the expected value model based on the work of Ouyang et al. (1996). A question in Chang et al. (2006) is that the result may not be right in case of fuzzifying some parameters in the result obtained in random circumstances. Dutta et al. (2007) considered a continuous review inventory model and presented a mixture inventory model by treating the annual average demand as a fuzzy random variable and developed a model. But in this research, the authors fuzzyfied the expected value of a random value (i.e. annual average demand) directly and adopted the possibilistic mean value to rank fuzzy number, that is inconsequential.

In this paper, we explore control policies in inventory systems with the continuous review style, and characterize the lead-time demands, defective rates in different cycles as independent and identically distributed (iid) random variables, respectively. Moreover, the backorder rate, ordering cost, penalty cost per unit item and marginal profit per unit investment in different cycle are characterized as iid fuzzy variables, respectively. The rest of this paper is organized as follows. Section 2 contains the concepts and essential properties of fuzzy variable, fuzzy random variable and fuzzy random process. In Section 3, the mathematical model is established, and also some useful propositions are presented. In order to estimate the expected value of fuzzy expression and search the optimal values of order quantity, reorder point and lead time, a fuzzy simulation algorithm and an iterative algorithm are designed, respectively. Section 4 provides an illustrative numerical example.

## 2. Preliminaries

### 2.1. Fuzzy variable

The basic concepts and operation rules about fuzzy set can be found in Zadeh (1965). After introduced the concepts of fuzzy set, Zadeh defined the concepts of possibility measure and necessity measure and established possibility theory (Zadeh, 1978, 1979). Although possibility measure has been widely used, it has no self-duality property. Also necessity measure is the same so. But a self-dual measure like probability is absolutely needed in both theory and practice. Recently, Liu and Liu (2002) presented the credibility measure, which has self-duality property. Moreover, as a new branch of mathematics, credibility theory was initiated by Liu (2004) to study the behavior of fuzzy events, and has been applied to modelling fuzzy optimization problems (see (Liu, 2005, 2006)). A detailed survey on credibility theory can be found in Liu (2006).

Let $\Theta$ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of $\Theta$. For an element $A$ in $\mathcal{P}(\Theta)$, $\mathrm{Cr}\{A\}$ expresses the chance that fuzzy event $A$ occurs and is called a credibility measure (see (Liu, 2004)). In addition, the triplet $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ is called a credibility space, and a fuzzy variable is defined as a function from the credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ to the set of real numbers (see (Liu, 2004; Liu, 2006)).

Let $\xi$ be a fuzzy variable defined on $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$, $u$ and $r$ real numbers. Then the credibility of the fuzzy event $\xi \leqslant r$, is defined by

$$\mathrm{Cr}\{\xi \leqslant r\} = \frac{1}{2}\left(\sup_{u \leqslant r} \mu(u) + 1 - \sup_{u > r} \mu(u)\right). \quad (1)$$

It is clear that $\mathrm{Cr}\{\xi \leqslant r\} + \mathrm{Cr}\{\xi > r\} = 1$, i.e., the credibility measure is self dual. Conversely, if $\xi$ is a fuzzy variable, then its membership function can be derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re. \quad (2)$$

**Definition 1** (Liu & Liu, 2002). Let $\xi$ be a fuzzy variable on a credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$. Then the expected value $E[\xi]$ is defined as

$$E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{\xi \leqslant r\}\mathrm{d}r, \quad (3)$$

provided that at least one of the two integrals is finite. Especially, if $\xi$ is a positive fuzzy variable, then $E[\xi] = \int_0^{+\infty} \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r$.

**Definition 2** (Liu, 2002). Let $\xi$ be a fuzzy variable on a credibility space $(\Theta, \mathcal{P}(\Theta), \mathrm{Cr})$ and $\alpha \in (0,1]$. Then

$$\xi_\alpha^L = \inf\{x | x \in \Re, \mu(x) \geqslant \alpha\} \text{and} \xi_\alpha^U = \sup\{x | x \in \Re, \mu(x) \geqslant \alpha\} \quad (4)$$

are called the $\alpha$-pessimistic and $\alpha$-optimistic values of $\xi$, respectively.

**Definition 3** (Liu & Gao, 2007). The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be independent if

$$\mathrm{Cr}\left\{\bigcap_{i=1}^{m}\{\xi_i \in B_i\}\right\} = \min_{1 \leqslant i \leqslant m} \mathrm{Cr}\{\xi_i \in B_i\} \quad (5)$$

for any sets $B_1, B_2, \ldots, B_m$ of $\Re$.

**Definition 4** (Liu, 2004). The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be identically distributed if and only if

$$\mathrm{Cr}\{\xi_i \in B\} = \mathrm{Cr}\{\xi_j \in B\}, \quad i,j = 1,2,\ldots,m \quad (6)$$

for any sets $B$ of $\Re$.