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Myopic optimal policy for a multi-period, two-delivery-lead-times, stochastic inventory problem with minimum cumulative commitment and capacity

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ABSTRACT

This paper studies a single-product, multi-period, stochastic inventory problem that imposes the lower and upper bounds on the cumulative order quantity during a planning horizon and allows two delivery lead times. This model includes three features. The first one is that a buyer purchases a fixed capacity from a supplier at the beginning of a planning horizon and the buyer's total cumulative order quantity during the planning horizon is constrained with the capacity. The second one is that the buyer agrees to purchase the product at least a certain percentage of the purchased capacity during the planning horizon. The third one is that the supplier allows the buyer to order the product with two-delivery-lead-times. We identify conditions under which a myopic ordering policy is optimal. We also develop an algorithm to calculate the optimal capacity when the minimum cumulative order quantity commitment is a certain percentage of the capacity. We then use the algorithm to evaluate the effect of the various parameters on the buyer's minimum expected total cost during the planning horizon. Our computation shows that the buyer would benefit from the commitments and two-delivery-lead-times.

1. Introduction

In recent years, there has been a growing interest in how commitments and flexible supply policies can be applied to improve the supply-chain efficiency from both the academic community and the industry. A commitment is an agreement between a supplier and a buyer that the supplier will deliver to the buyer a product over time with amounts and intervals specified before the beginning of a planning horizon. Commitments have potential advantages for the supplier because they assure demand and collect partial revenue in advance. An advantage to both parties is that the commitment provides the supplier guidance on the amount needed by the buyer and so helps the supplier assure timely delivery.

Flexible supply policies have been demonstrated to be able to help supply chains respond rapidly to changing market conditions. One way to achieve the flexibility is by using alternate delivery lead times. There are potential benefits for a buyer because the buyer may experience higher demand than expected during a regular delivery lead time and the buyer may use an expedited delivery lead time to satisfy unexpected demand. The purpose of the paper is to develop a multi-period stochastic inventory model that includes commitments and alternate delivery lead times from a buyer's perspective. In the rest of the section, we review the contributions that are most relevant to our research.

Bassok and Anupindi (1997) study a multi-period supply contract with total minimum commitment. In this contract, a buyer commits to purchasing the product at least certain amount during a planning horizon. They study the optimal ordering policy. Tibben-Lembke (2004) extends this model by adding lower and upper bounds on the order quantity in each period. The author studies the optimal and heuristic ordering policies. Xu (2009b) studies a multi-period supply contract with capacity commitment. In this contract, a buyer purchases a fixed capacity from a supplier at the beginning of a planning horizon. The author studies the optimal ordering policy and the optimal capacity commitment.

Commitments may also be flexible, i.e., the commitments are allowed to be revised within specified limits as new information becomes available. Anupindi and Akella (1993) discuss models in which there is an initial commitment in each period during a planning horizon and commitments can only be altered in the current period by at most a given percentage. They provide heuristics to evaluate the performance of the commitment. Bassok and Anupindi (1998) discuss models that allow commitments to be altered in each period by at most a given percentage of the commitments in the preceding period and provide heuristics. Tsay and Lovejoy (1999) also study a model that allows commitments to be revised in each period by at most a given percentage of the commitments in the preceding

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period without penalties and provide heuristics. Moinzadeh and Nahmias (2000) discuss a heuristic for an upward adjustment contract in which there is a fixed initial commitment at regular time intervals and a buyer is allowed to increase the commitment prior to delivery. There is a fixed cost for making an upward adjustment and all the data are stationary. Chen et al. (2001) study a warehousing contract under which space usage commitments may be changed at a finite number of times per year. Urban (2000) studies a multi-period supply contract with periodical commitment, in which an order quantity in each period is fixed and stationary, with limited flexibility to change the order quantity at a cost to the buyer.

There are papers that study capacity commitments from both a buyer's and a supplier's perspectives. Serel et al. (2001) and Serel (2007) study the problem in which a buyer makes a fixed capacity commitment in each period before a planning horizon. They study the steady state performance under the contract from both the buyer's and the supplier's perspectives. Ferguson (2003) studies the problem in which a buyer has only one ordering opportunity from a supplier but he can place the order before or after he sees a demand signal. The author discusses when to place the order from both the buyer's and the supplier's perspectives. Zhao et al. (2007) study the problem in which a buyer can place an order early from a supplier. This ordering assumption implies that the buyer has a longer delivery lead time. They assume that both the buyer and the supplier use base-stock policies and study the performance from both the buyer's and the supplier's perspectives.

The study of myopic policies for multi-period, stochastic inventory problems dates back to the early days of the literature. (see Veinott (1965), Ignall and Veinott (1969), Heyman and Sobel (1984) and Porteus (2002)). The key assumption in their models is that there are no capacity constraints on an order quantity in each period. When there are capacity constraints on an order quantity in each period, Federgruen and Zipkin (1986a,b) characterize the structure of the optimal ordering policy when the objectives are, respectively, to minimize the average cost and discounted cost. That is, a modified base-stock policy is optimal. Tayur (1993) shows how to compute the optimal base-stock level efficiently for this problem when the objective is to minimize the average cost. However, a myopic policy is not necessarily optimal for the inventory problem with capacity constraints.

The other research related to our research is the alternate delivery lead times. The shorter the lead time for delivery of a product, the lower will be the buyer's minimum expected cost—provided the ordering cost is independent of the lead time (Veinott (1998)). Short or long lead times may be more attractive according as inventories are low or high. Fukuda (1964) studies this problem for the case in which there are two possible delivery lead times that differ by only one period. The author shows that the optimal ordering policy in a period is described by two base-stock levels. Sethi et al. (2003) study the same problem but with fixed ordering cost and regular demand forecast. They characterize the structure of the optimal policy. Xu (2006) studies a multi-period stochastic inventory problem with option and capacity constraint. The option can be used to purchase a product with a shorter delivery lead time. The author characterizes the structure of the optimal option and ordering policy. Xu (2009a) studies a two facility supply chain with two-delivery-lead-times and storage limitation in both facilities. The author characterizes the structure of the optimal ordering policy for the supply chain.

For the case in which two delivery lead times differ by an arbitrary positive integer, Whittemore and Saunder (1977) derive the optimal ordering policy. However, the structure of the optimal policy is too complex and difficult to implement in practice. Thus, the focus of the research is to develop heuristics for this case. Zhang and Hausman (1994) discuss heuristics for the case in which there are two delivery lead times that differ by an arbitrary positive integer. Scheller-Wolf et al. (2003) also discuss heuristics for the same case but with service constraints.

In this paper, we study a single-product, multi-period, stochastic inventory problem that imposes lower and upper bounds on the cumulative order quantity and allows two delivery lead times from a buyer's perspective. The main contributions of the paper are as follows:

1. Our model significantly differs from the literature. Three papers by Bassok and Anupindi (1997), Xu (2009a, b) are most related to the research. Bassok and Anupindi (1997) study a multi-period stochastic inventory model with a minimum cumulative order quantity commitment during a planning horizon. Xu (2009b) studies a multi-period stochastic inventory model with a capacity commitment during a planning horizon. Xu (2009a) studies a multi-period two-facility stochastic supply chain with storage limitations at both facilities. Our model extends the models developed by Bassok and Anupindi (1997) and Xu (2009b) in two ways. One is that our model imposes both the lower and upper bounds on the cumulative order quantity during a planning horizon. The other is to allow two delivery lead times. The differences between our model and the two delivery lead times model developed by Xu (2009a) are two ways. One is that our model imposes upper and lower bounds on the cumulative order quantity while the model by Xu (2009a) imposes the storage limitations at both facilities. The other one is that the bounds in our model are decision variables and have to be determined at the beginning of a planning horizon while the storage limitations are input data in Xu's model (2009a).
2. We identify the conditions under which a myopic policy is optimal for the multi-period stochastic inventory problem that imposes the lower and upper bound on the cumulative order quantity during a planning horizon and allows two delivery lead times.
3. For the case that the lower bound commitment on the cumulative order quantity is a certain percentage of the upper bound commitment (capacity) on the cumulative order quantity, we develop an algorithm to calculate an optimal capacity. We then use the algorithm to evaluate the performance of the model and to show that the buyer may benefit from the commitments and two-delivery-lead-times.

The rest of the paper is organized as follows: the next section formulates the buyer's optimization problem. We assume that the expedited delivery lead time is zero period and the regular delivery lead time is one period. This assumption is carried out in Sections 3 and 4. Section 3 identities the conditions under which a myopic ordering policy is optimal. We then develop an algorithm to calculate the optimal capacity commitment if the lower bound commitment is a certain percentage of the capacity commitment. Section 4 uses simulation to evaluate the effect of various parameters on the buyer's minimum expected total cost during a planning horizon. Section 5 extends the results obtained in Section 3 to the case where the regular delivery lead time is a positive integer number of periods and the expedited delivery lead time is one period less than the regular one. This section also provides conclusions.

2. Model formulation

Consider a buyer who wishes to satisfy independent, identical, multi-period and stochastic nonnegative demands $D_1, D_2, \ldots, D_N$ for a product in periods $1, 2, \ldots, N$ with known cumulative probability distribution Φ. At the beginning of a planning horizon, the buyer purchases a fixed capacity for the planning horizon from a