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Review

Games with incomplete information: A simplified exposition with inventory management applications

H. Wu, M. Parlar *,1

DeGroote School of Business, McMaster University, Hamilton, Ontario, Canada L8S 4M4

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ABSTRACT

In most existing literature in supply chain management it is assumed that the players possess complete information about the game, i.e., the players' payoff (objective) functions are assumed to be common knowledge. For static and dynamic games with complete information, the Nash equilibrium and subgame perfect equilibrium are the standard solution concepts, respectively. For static and dynamic games with incomplete information, the Bayesian Nash equilibrium and perfect Bayesian equilibrium, respectively, are used as solution concepts. After presenting a brief review of the static and dynamic games under complete information, the application of these two games in inventory management is illustrated by using a single-period stochastic inventory problem with two competing newsvendors. Next, we illustrate the Bayesian Nash and perfect Bayesian equilibrium solution concepts for the static and dynamic games under incomplete information with two competing newsvendors. The expository nature of our paper may help researchers in inventory/supply chain management gain easy access to the complicated notions related to the games played under incomplete information.

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* Corresponding author.
E-mail address: parlar@mcmaster.ca (M. Parlar).
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1. Introduction

Game theory studies multiple-person decision problems involving conflict or cooperation. Following the publication of von Neumann and Morgenstern's (1944) seminal book, interest in potential applications of game theory reached a peak in the following decade. The fundamental solution concepts of game theory (e.g., the Nash equilibrium for non-cooperative games, Nash (1950), and the Shapley value for cooperative games, Shapley (1953)) were developed in the 1950s and were used to analyze problems in diverse areas including economics, political science, management-labor arbitration, philosophy and warfare.

Early applications of game theory considered games of "complete information", where each player's payoff (objective) function is common knowledge for all players. However, the stringent (and unrealistic) assumption of complete information became a barrier to successful implementation of game theoretic ideas because in most competitive problems the players are not privy to each other's payoff functions. For example, two firms competing for the same market demand do not have complete information on each other's production cost functions. Similarly, in a sealed-bid auction, the bidders do not know each other's valuations. In the late 1960s, Harsanyi (1967) developed solution concepts for games with incomplete, i.e., asymmetric, information (also known as the Bayesian games). In such games, players' payoff functions are no longer common knowledge; instead, at least one player is uncertain about another player's payoff function. With Harsanyi's discovery of the new solution concepts for incomplete information games, interest in game theory was heightened in the last two decades and game theory once again became an important tool that can be used to analyze realistic problems of competitive situations.

Operations researchers were early users of game theory as can be seen in the operations research texts published in the 1950s and 1960s. The textbooks by Churchman et al. (1958), Sasieni et al. (1959), Hillier and Lieberman (1986) and Ackoff and Sasieni (1968), all include a chapter on competitive problems. All four texts cover zero-sum games and all, except Hillier and Lieberman (1986), present a few examples of non-zero sum games involving bidding strategies. Shubik (1955) reviewed early publications in this area. However, after the initial excitement generated by its potential applications, operations researchers' interest in game theory seemed to have waned during the 1970s and the 1980s. But the last two decades have witnessed a renewed interest by academics and practitioners in the management of supply chains and a new emphasis on the interactions among decision makers ("players") constituting a supply chain. This has resulted in the proliferation of game theoretical publications in operations research/management science/operations management (OR/MS/OM) journals dealing with the use of game theory in the competitive and cooperative problems arising in supply chain management (SCM). For an excellent review of game theoretic applications in supply chain management we refer the reader to Cachon and Netessine (2004); see also a more recent review by Leng and Parlar (2005).

In their respective reviews of game theory applications in SCM, Cachon and Netessine (2004) and Leng and Parlar (2005) each cite more than 100 papers. It is interesting to note that a large majority of the reviewed papers make the simplified (and frequently unrealistic) assumption that all players know each other's objective functions with certainty. That is, they investigate problems dealing with games under *complete information*. Cachon and Netessine (2004) briefly mention signaling, screening and the Bayesian games where the games are played under *incomplete information*, i.e., at least one of the players does not know the other players' objective function. As examples of games played under incomplete information, they cite Cachon and Lariviere (2001) who applied a signaling game to study a contracting problem with information sharing in a one-supplier, one-manufacturer supply chain, and Cachon and Lariviere (1999) who studied a capacity allocation problem with information sharing issue between a supplier and several downstream retailers. Contract design problems also involve games of incomplete information; one of the earliest papers in this area is by Corbett (2001) who applied the principal-agent theory to design an inventory contract in the context of the (Q,r) model. In a more recent paper, Chu and Lee (2006) studied an information sharing problem in a vertical supply chain with one vendor and one retailer and employed the perfect Bayesian equilibrium as the solution concept used in dynamic games played under incomplete information. In a recent paper, Costantino and DiGravio (2009) combine concepts from game theory and fuzzy logic to analyze a bargaining problem with incomplete information.

The reviews by Cachon and Netessine (2004) and Leng and Parlar (2005) reveal that there is a paucity of papers that deal with games played under incomplete information. However, in recent years publications have begun to appear that analyze games played under incomplete information. Since most realistic SCM problems involve competitive interactions with incomplete information, it would be useful to provide an exposition of such games with applications to a specific area in SCM, namely, inventory management. With this in mind, we wrote this paper to present a simplified treatment of games with incomplete information with applications in stochastic inventory management.

We follow the same framework as in Gibbons (1992, 1997) who has also considered static and dynamic *complete* and *incomplete* information games and their applications in economics. Gibbons's classification results in four categories: (i) static games with complete information (for which the solution concept used is the Nash equilibrium), (ii) dynamic games with complete information (subgame perfection and the Stackelberg equilibrium), (iii) static games with incomplete information (the Bayesian Nash equilibrium), and (iv) dynamic game with incomplete information (perfect Bayesian Nash equilibrium). We start by briefly describing static and dynamic *complete* information games. This is followed by a more detailed exposition of static and dynamic *incomplete* information games. We first illustrate each of the four cases (which we call "Models") with a simple discrete game where each player has two moves. For each case, we then present a single-period stochastic inventory game with two competing newsvendors with the players' decision variables as continuous values. While our paper is expository in nature, it also contributes to the literature by presenting explicit methods for dealing with static and dynamic inventory games under incomplete information and computing the Bayesian Nash and perfect Bayesian equilibrium for such games. For another, and very detailed, treatment of static and dynamic complete and incomplete information games and their applications in economics; see, Fudenberg and Tirole (1992). Myerson (1991) and Osborne and Rubinstein (1994) also provide excellent overviews of game theory with economic applications.