Int. J. Production Economics 134 (2011) 123–137

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Inventory management with random supply and imperfect information: A hidden Markov model

Kenan Arifoğlu, Süleyman Özekici *

Department of Industrial Engineering, Koç University, 34450 Sarıyer-İstanbul, Turkey

ARTICLE INFO

Article history:
Received 30 January 2008
Accepted 28 April 2011
Available online 25 June 2011

Keywords:
Random supply
Random environment
Imperfect information
Base-stock policy
Dynamic programming
Sufficient statistics
POMDP

ABSTRACT

In most of the papers on inventory models operating in a random environment, the state of the environment in each period is assumed to be fully observed with perfect information. However, this assumption is not realistic in most real-life situations and we provide a remedy in this paper by assuming that the environment is only partially observed with imperfect information. We accomplish this by analyzing two formulations of single-item models with periodic-review and random supply in a random environment. In the first one, supply is random due to random capacity of production and random availability of transportation. We show that state-dependent base-stock policy is optimal if the capacity and all costs are observed, while demand and availability are unobserved. In the second model, we consider a model with random availability only with fixed-ordering cost. We show that state-dependent (s,S) policy is optimal if the availability process is observable.

1. Introduction

In any inventory system, there are several sources of uncertainty and the randomness in customer demand has always been the primary source. However, the supply may also be random and this represents a rather new paradigm in inventory management. In a recent article, Serel (2008) considers a single-period problem in the presence of two alternative suppliers one of whom may not always be available. He also presents a strong case for inventory models that incorporate supply uncertainty in decision making. There are two types of uncertainty on the supply side. First, the producer's capacity may vary due to numerous reasons such as the unavailability of machinery or workforce for varying periods of time due to unplanned maintenance (Wang and Gerchak, 1996) or production of defective items during the production process (Ciarallo et al., 1994). We model this first type of uncertainty via random production capacity. Second, the entire production may cease due to planned maintenance of the machinery used to produce the product (Ciarallo et al., 1994) or due to unexpected events such as fire, earthquake, strike or unplanned maintenance of the entire production facility (Hopp et al., 2010). This second type of uncertainty can also be modeled as random transportation availability The logistics firm may not be able to deliver the entire order due to accidents. This uncertainty type on the supply-side may significantly affect the performance of the entire supply chain and, hence, the inventory manager (IM) should consider them while determining the order quantity. For example, Norrman and Jansson (2004) state that a fire at a supplier's plant disrupted the supply of radio-frequency chips, resulting in a loss of $400 million in the spring of 2001. Juttner (2005) reports that in the same year, the continuity of production at Land Rover was threatened due to financial problems faced by the UK chassis manufacturer UPF Thompson. Kharif (2003) states that Motorola failed to ship the phones promised to its major customers during the holiday season in 2003 due to component shortages. These and other types of uncertainties may be modeled via a combination of random capacity and random availability in this paper.

Starting from 1950s, a number of research articles focused on the impact of the supply-side on uncertainty. Karlin (1958a,b) observed the fact that the quantity received is not necessarily equal to quantity ordered. Research considering the randomness in supply increased further after late 1970s. An earlier review about inventory models with random supply is provided in Yano and Lee (1995) and recent advances are summarized in Grosfeld-Nir and Gerchak (2004). In this paper, we aim to present rather general and realistic models of supply uncertainty or risk. In this respect, we bring random capacity and random availability models in the literature together. Random capacity models that are closely related to ours are those of Federgruen and Zipkin (1986) and Ciarallo et al. (1994). In addition, there are papers modeling randomly available suppliers in the literature. Parlar et al. (1995) consider a periodic-review inventory model with set-up costs where an order is either filled in full or not at all with some probability, and this probability depends on whether the supply was available in the

* Corresponding author. Tel.: +90 212 3381723; fax: +90 212 3381548.
E-mail address: sozekici@ku.edu.tr (S. Özekici).

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.04.033

previous period. They show that the optimal policy is of (*s*,*S*) type, where *s* depends on supplier's state in the last period. Another related paper is Güllü et al. (1999) who analyze a periodic-review, single-item inventory model where supply is either fully available, partially available or unavailable. At the end, they show that base-stock policy is optimal. Moreover, Ertogral and Rahim (2005), and Chiang (2008) also assume that the supplier is randomly available; however, unlike previous papers, they suppose that the review period is not fixed but random due to random visits by the supplier. Yan and Liu (2009) present a periodic-review, finite-horizon inventory model in which there are two suppliers who charge fixed-ordering cost, and the IM decides the order quantity from each as well as the sales price. One supplier is expensive, but also fast and reliable, whereas the other is cheap, but delivers the order in the next period in full or not at all. Our paper is also related to the growing literature (Serel, 2008; Kelle et al., 2009; Li et al., 2010) modeling contractual relationships and risk sharing to decrease operational risk due to random supply.

Another important feature of our analysis concerns the affect of the environment. Majority of the literature is on models where the inventory system operates in a stationary environment. However, inventory systems are quite sensitive to changes in the surrounding environment consisting of economic, financial, social, political, and other factors which affect demand, supply and all cost parameters. This is evident with the current state of the world economy in a recession that results in decreased demand and increased supply uncertainty. The prevailing economic conditions of the environment change randomly and they have their impact on all stochastic and deterministic model parameters. For more realistic inventory models, one must consider the possible effects of changing economic conditions, market conditions and exogenous environmental factors on demand, supply and all costs. Demand for certain products is subject to significant changes throughout product life cycle and a stationary demand structure is often unrealistic. In this respect, Karlin and Fabens (1959) and Iglehart and Karlin (1962) were among the first to consider fluctuations in the demand environment. Later, random environmental processes were used in a number of papers including Song and Zipkin (1993), Song and Zipkin (1996), Sethi and Cheng (1997), Özekici and Parlar (1999), Erdem and Özekici (2002), Gallego and Hu (2004), and Mohebbi (2006). In periodic-review models, the environmental process is generally taken to be a Markov chain as it is the case in our present setting.

Most inventory models with a random environment suppose that observations of the inventory manager (IM) regarding the true state of the environment is perfect. However, observations may be partial or incomplete leading to imperfect information on the environment. These observations are not complete since all known and unknown factors defining the environment are subject to fluctuations. In other words, observations are imperfect because it is impossible either to identify the complete set of factors defining the environment, or to follow their fluctuations. For example, there is often confusion as to when, or if, an economy has entered recession or another undesirable state. Although not perfect, the observations give partial information about the true state of the environment. Therefore, we model the random environment by a hidden process that is not perfectly observable and suppose that a process which gives partial information about the real environmental process is observed. The IM continues to make decisions even though the true state of the environment is unknown and the decisions should be based on the observed process. Our primary assumption and contribution is that the environmental process is given by a so-called hidden Markov chain that is not completely observed by the IM. Processes of this type, where the random environment is represented by the Markov chain and the true state of this Markov chain cannot be observed directly (however, there is another process which gives partial information about the true state) are also called "partially observed Markov decision processes" (POMDP). POMDP have a wide range of application areas-like machine maintenance and replacement, human learning and instruction, medical diagnosis and decision-making, among many others. Although there is an extensive research on POMDP, they are not employed frequently in the inventory literature. In their paper, Treharne and Sox (2002) represent the demand environment by the Markov chain whose state is only partially observed. They assume that the capacity of the supplier is infinite and show that a state-dependent base-stock policy is optimal where the state is the inventory position and the conditional distribution of the true environmental state. Another paper which applies POMDP in inventory management is Bensoussan and Çakanyıldırım (2005) who study three different models: information delay, filtered newsvendor and zero balance walk. They show that the state-dependent base-stock policy is optimal for information delay model, whereas optimal feedback policy is optimal for the other two models. In a similar paper, Arifoğlu and Özekici (2010) study an inventory system with random yield and finite capacity operating in a partially observed random environment and show that the state-dependent modified inflated base-stock policy is optimal.

In this paper, our main contribution is to characterize the optimal ordering policies for inventory problems in very general settings where the demand and supply are random, and they are modulated by a partially observed random environment together with all cost parameters. This is done for a couple of models. In the first one, we assume that there is no fixed-ordering cost, and that there is a supplier with random capacity and a transporter with random availability. We show that the state-dependent base-stock policy is optimal if the capacity process is observable; otherwise, the optimal ordering policy has a rather complicated structure. In the second one, there is a fixed-ordering cost and we suppose that the supplier has infinite capacity. We show that the state-dependent (*s*,*S*) policy is optimal for these inventory problems. Since the model is modulated by an external process that is not perfectly observable, the decisions of the IM is affected by the imperfect observations on the state of this process. We also discuss the implications of information precision on managerial decisions. Our numerical analysis indicates that the IM makes better decisions as the precision in the observations increases, and that the IM carries more inventory and orders less frequently as demand uncertainty increases and supply reliability decreases.

The rest of the paper is organized as follows. In Section 2, we describe our model in detail. Then, in Section 3, we analyze an inventory model with random capacity and random availability in a hidden random environment when there is no fixed-ordering cost. We suppose that the supplier's capacity is infinite but there is random availability in Section 4 and assume that there is fixed-ordering cost. Section 5 includes a numerical illustration that demonstrates how our results can be used where we also discuss the effect of the precision in our observations on the optimal policy. Finally, in Section 6, we conclude and present a discussion on possible extensions. Technical material including derivations and detailed proofs of some results can be found in the Appendix.

2. Model description

We consider a single product inventory system in a random environment which is observed periodically over a planning horizon of duration N. The true state of the environment at time n, denoted by Z_n, form a time-dependent Markov chain $Z=\{Z_n;\ n=0,1,2,3,\ldots\}$ on a discrete state space $\mathbb{F}=\{a,b,c,\ldots\}$ with transition matrix $Q_n(a,b)=P[Z_{n+1}=b|Z_n=a]$. The IM does not have