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An inventory model with random discount offerings

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A B S T R A C T

We consider an inventory model in which a supplier makes deal offers with random discount prices at random points in time. Assuming that discount offerings follow a Poisson process and discount price is a discrete random variable with a known distribution, we propose a continuous-review control policy for the model and derive optimality conditions for the policy parameters. The model is then extended to the case of multiple suppliers that offer discount deals with supplier-specific Poisson processes and discount prices. Numerical examples are presented to demonstrate cost savings due to discount offers.

1. Introduction

In many real world applications, suppliers offer occasional price discounts. Such discount offers can be due to market imbalances and excessive inventory in the marketplace or at the suppliers [1]. Examples are abundant in grocers and supermarket chains [2]. Another example is the Sony Company that has resorted to frequent price-discount offerings on its LCD rear-view projection sets in order to prevent losing sales and profits to no-name Chinese rivals [3].

Inventory management with uncertain prices has received intense attention in the operations research and management science community for years. Some of the works in the literature have incorporated price fluctuation in inventory models as a one time price change at some pre-announced time in the future [4–8], while others have considered on-going price fluctuations in the inventory management problem [9–11]. The inventory replenishment decision upon randomly arriving discount prices in a continuous-review inventory system has been studied [12–16,1].

Friend [12] studies a continuous-review inventory model with Poisson demand, where lead times are negligible and opportunities for economic replenishments are offered according to a Poisson process. In his model, opportunity replenishment does not represent a lower unit purchase cost but only a reduction or elimination of the fixed ordering cost of a replenishment. Hurter and Kaminsky [14] study a similar problem, where price discounts are offered according to a Poisson process. Hurter and Kaminsky [13] extend the storage model of Friend [12] and Hurter and Kaminsky [14] to permit the opportunities for a discounted purchase to remain for a random length of time when they arise; hence, the opportunities to purchase at a lower price are no longer instantaneous. Silver et al. [15] develop an efficient approximate solution method for a special case of the model analyzed by Hurter and Kaminsky [14] and provide managerial insights on the behavior of the optimal policy parameters. Feng and Sun [17] derive the form of the optimal replenishment policy for the model of Hurter and Kaminsky [14]; that is, they provide the policy that minimizes the long-run average cost of inventory systems with Poisson demand and discount opportunities. To find the parameters of this optimal policy, Feng and Sun [18] develop an efficient algorithm based on a bisection search procedure.

Moinzadeh [1] again analyzes the model of Hurter and Kaminsky [14] but with a constant demand rate. In particular, he assumes that deal (price discount) offerings follow a Poisson process and that the deal offers have no duration. He derives expressions for determining the optimal policy parameters and provides results on their properties. Goh and Sharafali [16] incorporate a flexible pricing policy in the model of Moinzadeh [1]. Their proposed pricing policy is inventory dependent, setting a lower price for higher inventory levels and the original price for lower inventory levels. Finally, Chaouch [3] considers a model similar to that of Hurter and Kaminsky [13], where discount opportunities are not instantaneous. That is, the low (discount) price goes into effect at random points in time and is increased to its original level after a random duration. He, however, allows the length of the low-price and high-price periods to be different. Moreover, he assumes a constant demand over time.

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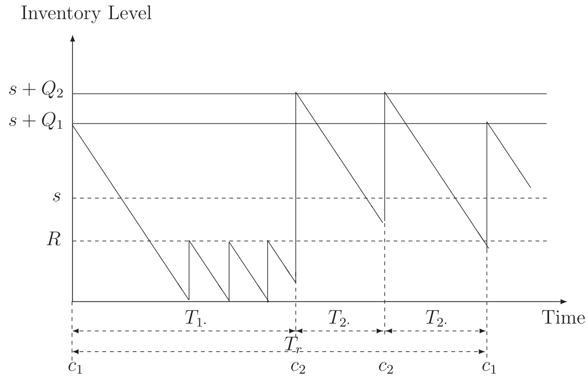

Fig. 1. Case 1 ($R \le s$).

Unlike Goh and Sharafali [16], he allows the demand rate in effect in a low-price period to be different from the one in a high-price period. He highlights an inventory control policy similar to that of Hurter and Kaminsky [13], and analyzes the behavior of the optimal replenishment strategy.

A major shortcoming of the current literature on inventory models with random discount offers is that the discount price, from a single supplier, is known a priori and deterministic, and just the discount offer arrivals are allowed to be random (e.g., see [1,14]). In reality the discount price itself may be a random variable whose distribution could be derived from historical data. In this paper, we consider a continuous-review inventory model in which deal offers arrive from a supplier in a Poisson fashion and discount prices of such offers can be represented by a discrete random variable with a known distribution. The discount opportunities are instantaneous; moreover, two different fixed ordering costs are considered for the regular and discount replenishments. We propose a control policy that answers the following questions: (1) "When should a discount offer be accepted?" (2) "When accepting a discount offer, how much should be ordered?" and (3) "When making a replenishment at the regular price, how much should be ordered?" We first develop optimality conditions for the policy parameters for the case of a single supplier. Then, assuming suppliers are differentiated only by their discount price distributions and deal offer rates, we propose a way to apply the results to the case of multiple suppliers. This generalization is supported by the fact that superposition of multiple independent Poisson processes is a Poisson process [19].

The main contribution of our paper is twofold. While all the models in the literature assume that only discount offer arrival is random, we allow the *discount price* itself to be also a general random variable as would be the case in reality. Furthermore, in contrast to the existing literature in which random discount opportunity is modeled in a single-sourcing setting, we introduce the *multiple-sourcing* practice in this literature.

This paper is organized as follows. Section 2 describes the mathematical model, the assumptions, and the policy employed. In Section 3, we analyze the model and derive the expected cost functions. In Section 4, we present important properties of the proposed policy and provide the optimal policy parameters. Section 5 explains how to use our inventory model in the presence of multiple suppliers. In Section 6, we give numerical studies to illustrate the advantages of random discount offers. Section 7 concludes the paper.

2. The model

Consider a continuous-review inventory system for a single product with a constant demand rate D. There is *a single supplier* who offers random discounts at random points in time. Specifically, the supplier offers discounts according to a Poisson process with a rate λ and the discount price offered can be one of n possible prices with a known distribution. Let C_D be a discrete random variable representing the discount price and let p_i be the probability that the discount price offered is $c_i \in \{c_1, c_2, \ldots, c_n\}$ such that $\sum_{i=1}^{n} p_i = 1$. Therefore, conditional on the arrival of an offer, we can write $\Pr\{C_D = c_i\} = p_i, i = 1, 2, \ldots, n$. The product can always be procured at a regular price c_L from the supplier, where $c_L > c_i$ for $i=1, 2, \ldots, n$. While replenishment lead times are negligible, an order at a discount price incurs a fixed cost of A_D and at the regular price incurs a fixed cost of A_L. We do not assume any relation between A_L and A_D. The unit holding cost per unit time is h and no backorders are allowed. In the same spirit as [1], we assume that the discount offer evaporates quickly so that the inventory manager has to make a decision instantly upon offer. Moreover, we assume that there is no warehouse capacity constraint, i.e., enough warehouse space is always available for the item.

We employ a replenishment and stocking policy similar to those of Hurter and Kaminsky [14] and Moinzadeh [1]. That is, we consider a price dependent two-bin policy, denoted as (R, s, Q_1, $Q_2, \ldots, Q_n$). When a discount offer is made with discount price c_i, if the inventory level is less than or equal to a threshold level "s", then a replenishment order is placed to raise the inventory to a discount price-specific target "$s+Q_i$". If the inventory is depleted and no discount is offered, then the inventory manager orders "R" units at the list (or regular) price. Notice that a list price purchase happens only when the inventory level hits zero. Therefore, the inventory policy of our interest consists of $(n+2)$ decision parameters: R, s, Q_1, $Q_2, \ldots,$ and Q_n. Depending on the relative magnitude of R and s, we have to consider two cases: Case 1, where $R \le s$, and Case 2, where $R > s$. Examples of the inventory profiles of the two cases are shown in Figs. 1 and 2. In these figures, $T_{i\cdot}$ $(i=1,2)$ represents the duration of state i (starting when an offer with discount price c_i is accepted and ending when another discount offer is accepted) and T_r represents the length of a cycle (the time between two consecutive discount replenishments at price c_1).

While the optimality of a special case of our policy studied in [1] remains unanswered, our policy appears to be suboptimal due to the fact that there is a single threshold value, s, for n different discount prices. Intuitively, we should decrease the threshold for a high discount price so that the high discount price is less likely to be accepted. Our proposed policy does not differentiate different discount prices in terms of the threshold, although it penalizes offers with higher discount prices by allocating smaller Q_i's. Incorporating different thresholds into the model would make the analysis and implementation of the

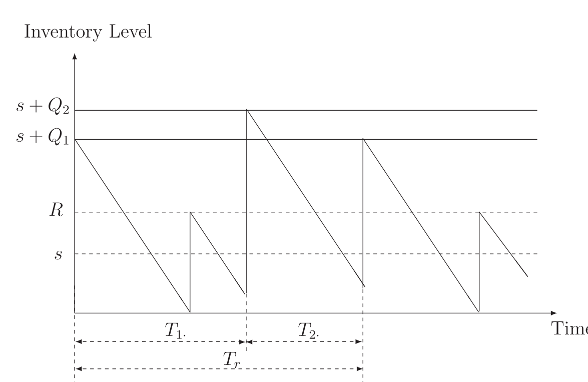

Fig. 2. Case 2 ($R > s$).