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Joint determination of inventory replenishment and sales effort with uncertain market responses

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ABSTRACT

An optimal joint operational and marketing decision is crucial for robust supply chain management. This paper addresses concurrent determination of inventory replenishment and sales effort decisions such as price, incentives to salesforce, and short-term promotions, or a combination of them. Market responses to sales efforts are typically highly uncertain, and demand in each period has its distribution dependent on the selected sales effort. In each period a replenishment order may be issued, which incurs both fixed and variable ordering costs, and at the same time the sales effort is also determined, the execution of which may incur costs. For such a model, the previously developed methods which are used for the joint inventory-pricing models become inadequate. A computational procedure for obtaining an optimal joint policy is addressed, and the conditions for the optimality of that policy are identified.

1. Introduction

Demand uncertainty is one of the most challenging but important issues in supply chain management. Traditional stochastic demand models are often assumed to be determined exogenously. Models taking price into consideration formulate demand with its distribution parameterized and controlled by the endogenous decision of price. The impact of price on demand is often either in an additive or a multiplicative form, or occasionally in a combination of both. In the additive form, demand is the summation of a deterministic component which is price-sensitive, and a random component which is independent of price, while in the multiplicative form, demand is modeled as a product of the two components (Yano and Gilbert, 2004).

In the retail setting, however, such uncertainty is compounded because demands depend on the marketing decisions such as price, efforts exerted by sales personnel, advertising campaigns, and competitions. In this paper, we focus on sale effort. Firms exert various sales efforts, and often at a combination of them to increase sales and profits. For example, price changes, incentives to the sales people, and promotions with short-term effects such as in-store displays and product-service bundling. Clearly, the relationship between the market response and the combination of these sales efforts can be much more complex as compared to a single one.

In this paper we allow a general relationship of the sales efforts and the market responses without specifying any form. Under such a setting, our interests lie in how to coordinate inventory and sales effort decisions. Specifically, we consider a single item, periodic-review system as follows. At the beginning of each period, the firm decides the order replenishment and sales effort jointly. Ordering incurs both fixed and variable costs, so does the execution of the sales effort, e.g., rentals for in-store displays, and advertisement costs for newspaper promotion. All replenishment arrives immediately, i.e., the leadtime is zero, and all unmet demand is fully backlogged. The inventory holding and shortage cost is charged based on the inventory leftover at the end of the period. The objective is to find a joint optimal inventory replenishment and sales effort policy to maximize the long-run average profit over an infinite horizon.

For such a model, the previously developed methods which are used for the joint inventory-pricing models become inadequate. It is therefore difficult to characterize the optimal inventory and sales effort policy. One of the joint policy forms is thus of special interest, the (s,S,z) form. Under an (s,S,z) policy, whenever the inventory level falls to or below s, an order is placed to bring it up to S; when the inventory level is at x, i.e., $s<x\le S$, no order is issued. The choice of sales effort z is determined based on the inventory level x. In other words, the stock is replenished according to a min–max ordering policy, and the decision on the sales effort is specified by the inventory level. This

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policy is a generalization of the popular (s,S,p) form, if the sales effort decision is replaced with the price decision. The task of this paper is thus twofold: to develop an efficient approach for searching for the optimal (s,S,z) policy; and to identify the optimality conditions under which the (s,S,z) policy is globally optimal.

Our model is closely related to the literature on the coordination between marketing and production/inventory management in general and to that on multi-period stochastic pricing and inventory control in particular. Here we briefly review the most relevant work, beginning with the literature on joint pricing and inventory control. Interested reader can refer to Yano and Gilbert (2004) for detailed reviews. For single period models, see, for example, Karakul (2008), Serel (2008), and Webster and Weng (2008). In a multi-period setting, when there is no ordering setup cost, Federgruen and Heching (1999) show that the pricing/inventory decision is the ‘‘base stock list price policy’’, which generalizes or extends many of the earlier models.

Recently, several papers have considered models with fixed ordering costs and identified the conditions under which an (s,S,p) policy is optimal. One key condition is related to a newsvendor-type profit function, which is defined as the resulting expected one-period profit with price being optimized for every inventory level. In a finite horizon periodic-review setting, Chen and Simchi-Levi (2004a) show the optimality of the (s,S,p) policy or a variation of such a policy. Chen and Simchi-Levi (2004b) extend the optimality of a stationary (s,S,p) policy to an infinite horizon setting. Both models require the newsvendor-type profit function to be concave in the inventory level. With a discounted profit criterion, Huh and Janakiraman (2008) further allow the newsvendor-type profit function to be unimodal. Feng and Chen (2004) propose a computational approach to calculate the optimal joint inventory-pricing control policy in an infinite-horizon periodic-review system, assuming that the newsvendor-type profit function is quasi-concave. However, such a condition can easily be violated in the current setting of our paper. Assuming unmet demands are lost, an earlier work of Polatoglu and Sahin (2000) analyzes a similar framework and characterizes the structure of optimal policy which is very complicated, for example, there may be multiple order-up-to levels. Though some sufficient conditions are provided under which the simple (s,S,p) policy is optimal, these conditions are difficult to verify and non-intuitive.

There are also some studies investigating the impact of various sales efforts on operational inventory decisions. To name a few, Balcer (1983) for the joint inventory and advertising strategy problem, Cheng and Sethi (1999) for the joint inventory-promotion problem with Markov-dependent demand state, Porteus and Whang (1991) and Chen (2000) for the impact of incentive schemes of salesforce compensation on manufacturing decisions. Ernst and Kouveils (1999) study the joint decision of goods bundling and inventory control in a newsvendor setting. In a continuous-review setting, Chen et al. (2005) show that the $(s,S)-$type policy is optimal for product inventory control, and an inventory level-based service package composite is optimal for service offerings. Zhang et al. (2008) discuss the joint optimization of inventory and pricing, and promotion, and ignore the fixed ordering cost in their setting.

The rest of this paper is organized as follows. Section 2 describes the model and the long-run average profit function of a given stationary (s,S,z) policy. Section 3 proposes an efficient computing procedure to find an optimal (s,S,z) policy. Section 4 further identifies the conditions under which the optimal (s,S,z) policy is globally optimal. Section 5 reports numerical findings, and Section 6 concludes the paper. All technical proofs are included in the Appendix.

2. The model

Consider a firm that makes inventory and sales effort decisions jointly in every period of an infinite horizon. All cost parameters are assumed to be stationary. The choice of sales effort in each period is made from a given finite, ordered list $\{z^1,z^2,\dots,z^M\}$, and the impact of sales effort decisions is assumed to be on demand in the current period. Each option of choice could be a single or combinations of multiple sales effort levels. If employing two marketing tactics, for example, price and in-store display, the sales effort list is then constructed by enumerating the total possible combinations of price and display options.

For each period $t=1,2,\dots,\infty$, let $D_t(z^i,\varepsilon)$ be the demand in period t, where z^i is the selected sales effort decision, $z^i\in\{z^1,z^2,\dots,z^M\}$, and ε is a time invariant random term with known distribution. Here we do not specify the relationship of the sales effort and the demand. Demand takes integer values with probability mass function $\phi_j(z^i), i=1,2,\dots,M;\ j=0,1,2,\dots,\infty$, and demands in any two consecutive periods are assumed to be independent and stochastically identical if the sales efforts are set to be the same.

The sequence of events in a period is as follows: the inventory level is reviewed, an order is issued if needed and then is delivered immediately, the sales effort is determined, demand is observed and satisfied, and relevant costs and revenue are evaluated.

Unmet demand is backlogged and will be fulfilled once stock becomes available. We also assume that the revenue is received when demand occurs, regardless whether or not that demand is fully met. The expected revenue in a period is thus a function of sales effort z, i.e., $p_zE[D(z,\varepsilon)]$, where p_z is either determined by the choice of sales effort z or determined exogenously.

Each replenishment incurs a fixed ordering cost K, and a variable unit cost c. The inventory holding/shortage cost is charged based on the inventory level x at the end of period t, and denoted by $g(x)$. It is convenient to assume that $g(x)$ is convex in x and unbounded as $|x|\to\infty$. Typically, $g(x)$ is minimized at $x=0$. Nevertheless, we generally denote by x_0 a minimizer of $g(x)$. Given that the inventory level after order replenishment at the beginning of period t is y, and sales effort is z, the expected inventory holding/shortage cost at the end of period t is denoted by $G(y,z)=E[g(y-D(z,\varepsilon))]$, with $D(z,\varepsilon)$ the demand for the period.

Our model allows for a cost component $A(z)$ that arises from executing the sales effort, which is important yet often ignored in the literature. This cost may vary with the choice of sales effort z. In the consumer electronics industry, for example, the cost associated with in-store displays is different from that associated with a newspaper advertisement.

Without loss of generality, we charge the variable ordering cost c only when items are consumed. As in defining $G(y,z)$, denote the *gross profit* in a period as

$$B(y,z)=(p_z-c)E[D(z,\varepsilon)]-A(z)-G(y,z). \tag{1}$$

It is so called because the fixed ordering cost K is ignored here.

As will be seen, the following newsvendor-type profit function defined based on $B(y,z)$ is an important instrument in the analysis

$$f(y)=\max_{z\in\{z^1,\cdots,z^M\}} B(y,z). \tag{2}$$

For ease of exposition, we impose a mild assumption on $f(y)$ as follows.

Assumption 1. $f(y)$ has a finite number of local maxima.

Assumption 1 can be trivially satisfied. Note that the sales effort z is selected from the finite list $\{z^1,\dots,z^M\}$, i.e., there are M available choices. For each z^i, $i=1,\cdots,M$, $B(y,z^i)$ is concave in y