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Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand

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ABSTRACT

In this paper, a joint pricing and inventory control for non-instantaneous deteriorating items is developed. We adopt a price and time dependent demand function. Shortages is allowed and partially backlogged. The major objective is to determine the optimal selling price, the optimal replenishment schedule and the optimal order quantity simultaneously such that, the total profit is maximized. We first show that for any given selling price, optimal replenishment schedule exists and unique. Then, we show that the total profit is a concave function of price. Next, we present a simple algorithm to find the optimal solution. Finally, we solve a numerical example to illustrate the solution procedure and the algorithm.

1. Introduction

Pricing and inventory policy are two important factors in success of business of different items. When the items are deteriorating the importance of these factors is increased. As declared in literature, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of commodity that results in decreasing usefulness. Most of physical goods undergo decay or deterioration over time, the examples being medicine, volatile liquids, blood banks, and others (Wee, 1993). Consequently, the pricing and inventory control problem of deteriorating items has been extensively studied by researchers. The first attempt to describe the optimal ordering policies for deteriorating items was made by Ghare and Schrader (1963). They presented an EOQ model for an exponentially decaying inventory. Later, Covert and Philip (1973) formulated the model by considering variable deteriorating rate with two-parameter Weibull distribution. Goyal and Giri (2001) provided an excellent and detailed review of deteriorating inventory literatures. Balkhi (2011) developed and solved a general finite trade credit economic ordering policy for an inventory model with deteriorating items under time value of money.

In some inventory systems, such as fashionable items, the length of the waiting time for next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment (Geetha and Uthayakumar, 2010). Abad (1996, 2001) considered a pricing and lot-sizing problem for a product with variable rate of deterioration, allowing shortages and partial backlogging. Dye (2007) developed a joint pricing and ordering policy for a deteriorating inventory with partial backlogging. The demand is known and linear function of price. Later, Dye et al. (2007) presented an inventory and pricing strategy for deteriorating items with shortages. Demand and deterioration rate are continuous and differentiable function of price and time, respectively. Chang et al. (2006) established an EOQ model for deteriorating items for a retailer for determine its optimal selling price and lot-sizing policy with partial backlogging and log-concave demand. Cai et al., (2011) studied qthe problem of pricing and ordering policy in two-stage supply chains by considering the partial lost sales and using the game theory.

In the deteriorating items inventory literatures that been mentioned above, all researchers assume that the deteriorating of the items in inventory starts from the instant of their arrival in stock. In fact, most goods would have a span of maintaining quality or original condition, namely, during that period, there is no deterioration occurring. Wu et al. (2006) defined the phenomenon as "non-instantaneous deterioration". They developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand such that the total relevant inventory cost per unit time had a minimum value. In the real world, this type of phenomenon exists commonly such as firsthand vegetables and fruits have a short span of maintaining fresh quality, in which there is almost no spoilage. Afterward, some of the items will start to decay. For this kind of items, the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to make inappropriate replenishment policies due to overvalue the total annual relevant inventory cost. Therefore, in the field of inventory management, it is

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necessary to consider the inventory problems for non-instantaneous deteriorating items.

Ouyang et al. (2006) present an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Although their inventory model is correct and interesting, processes of arguments to derive those theorems and the easy-to-use method to search for the optimal replenishment cycle time are not complete. Chung (2009) considers this issue and present complete proofs for Ouyang et al. (2006) model.

Yang et al. (2009) developed an inventory system for non-instantaneous deteriorating items with price-dependent demand. In their model, shortages are allowed and partially backlogged. The major objective is to determine the optimal selling price, the length of time in which there is no inventory shortage, and the replenishment cycle time simultaneously.

Geetha and Uthayakumar (2010) proposed EOQ based model for non-instantaneous deteriorating items with permissible delay in payments. In this model demand and price is constant and shortages are allowed and partially backlogged. Musa and Sani (in press) developed a mathematical model for inventory control of non-instantaneous deteriorating items with permissible delay in payments.

An appropriate pricing and inventory model for a non-instantaneous deteriorating item is presented in this paper. In the sum of all works on pricing and inventory control, models that consider non-instantaneous deteriorating items are very small portion. on the other side, in all papers that consider pricing and inventory control for non-instantaneous deteriorating items, the demand functions are simple and dependent on price, stock or time, separately. But in non-deteriorating items the price and the time should be considered jointly. Because, this form of demand function reflect a real situation: i.e. the demand may increase when the price decreases, or it may vary through time. In this work, we consider time and price dependent demand function. Also, Shortages are allowed and partially backlogged. The backlogging rate is variable and dependent on the waiting time for the next replenishment. The major objective is to determine the optimal selling price, the optimal replenishment cycle time and the order quantity simultaneously. This is the first work that has the above assumptions.

The rest of the paper is follows. In Section 2, assumptions and notations used throughout this paper are present. In Section 3, we establish the mathematical model and the necessary condition for finding an optimal solution. For any given selling price, we then show that the optimal solution is exist and unique. Moreover, we prove that the total profit is a concave function of selling price. Next, in Section 4, we present a simple algorithm to find the optimal selling price and inventory control variables. In Section 5, we use a numerical example and finally, we make a summary and provide some suggestions for future in Section 6.

2. Notations and assumptions

The following notations and assumption are used throughout the paper:

Notations:

c	the constant purchasing cost per unit
h	the holding cost per unit per unit time
s	the backorder cost per unit per unit time
o	the cost of lost sale per unit
p	the selling price per unit, where $p>c$
θ	the parameter of deterioration rate of the stock
t_d	the length of time in which the product exhibit no deterioration
t_1	the length of time in which there is no inventory shortage
T	the length of replenishment cycle time
Q	the order quantity
p^*	the optimal selling price per unit
t_1^*	the optimal length of time in which there is no inventory shortage
T^*	the optimal length of the replenishment cycle time
Q^*	the optimal order quantity
$I_1(t)$	the inventory level at time $t\in[0,t_d]$
$I_2(t)$	the inventory level at time $t\in[t_d,t_1]$
$I_3(t)$	the inventory level at time $t\in[t_1,T]$
I_0	the maximum inventory level
S	the maximum amount of demand backlogged
$TP(p,t_1,T)$	the total profit per unit time of the inventory system
TP^*	the optimal total profit per unit time of the inventory system, that is, $TP^*=TP(p^*,t_1^*,T^*)$

Assumptions:

- A single non-instantaneous deteriorating item is assumed.
- The replenishment rate is infinite and the lead time is zero.
- The basic demand rate, $D(p,t)=(a-bp)e^{\lambda t}$ (where $a>0$, $b>0$) is a linearly decreasing function of the price and decreases (increases) exponentially with time when $\lambda<0(\lambda>0)$. Thus, the demand rate is a function of price and time, which should reflect a real situation: i.e. the demand may increase when the price decreases, or it may vary through time. Here we adopt the form of the multiplicative exponential time effect for the basic demand rate, which is suitable for describing the time-varying demand. Given a different λ, which can be either positive or negative, this form can represent most cases where demand rate varies with time. The consideration of the time and price dependent demand is useful for the deteriorate items such as: fashion goods, high-tech product, fruits and vegetables (Tsao et al., 2008).
- Shortages are allowed. We adopt the notation used in Abad (1996) where the unsatisfied demand is backlogged, and the fraction of shortage backordered is $\beta(x)=k_0e^{-\delta x}$, $(0<k_0\le 1,\delta>0)$, where x is the waiting time up to the next replenishment and δ is a positive constant and $0\le\beta(x)\le 1,\beta(0)=1$. To guarantee the existence of an optimal solution, we assume that $\beta(x)+H(\beta'(x))>0$, where $\beta'(x)$ is the first derivative of $\beta(x)$. Note that if $\beta(x)=1$ (or 0) for all x, then shortage is completely backlogged (or lost).
- The length of time in which the product exhibit no deterioration is larger than or equal to the length of time in which there is no inventory shortage, i.e. $t_1\ge t_d$.

3. Model formulation

For simplicity, we use the same inventory shortage model as in Yang et al. (2009). Base on this model; the inventory system is as follows. I_0 units of item arrive at the inventory system at the beginning of each cycle and drop to zero due to demand and deterioration. Then shortage occurs until the end of the current order cycle.

We assume that the length of time in which there is no shortage is larger than or equal to the length of time in which the product has no deterioration. During the time interval $[0,t_d]$, the inventory level decreases due to demand only. Subsequently the inventory level drops to zero due to both demand and deterioration during the time interval $[t_1,t_d]$. Finally, a shortage occurs due to demand and partial backlogging during the time interval $[t_1,T]$.

During the time interval $[0,t_d]$, the differential equation representing the inventory status is given by

$$\frac{dI_1(t)}{dt}=-D(p,t)=-(a-bp)e^{\lambda t},\quad 0\le t\le t_d, \tag{1}$$