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Continuous review inventory models with a mixture of backorders and lost sales under fuzzy demand and different decision situations

Lin Wang [a,*], Qing-Liang Fu [a], Yu-Rong Zeng [b]

[a] School of Management, Huazhong University of Science & Technology, Wuhan, Hubei 430074, China
[b] School of Information Management, Hubei University of Economics, Wuhan, Hubei 430205, China

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ABSTRACT

In this paper, continuous review inventory models in which a fraction of demand is backordered and the remaining fraction is lost during the stock out period are considered under fuzzy demands. In order to find the optimal decision under different situations, two decision methods are proposed. The first one is finding a minimum value of the expected annual total cost, and the second one is maximizing the credibility of an event that the total cost in the planning periods does not exceed a certain budget level. For the first decision method, an approach of ranking fuzzy numbers by their possibilistic mean value is adopted to achieve the optimal solution. For the second one, the technique of fuzzy simulation and differential evolution algorithms are integrated to design hybrid intelligent algorithms to solve the fuzzy models. Subsequently, the two decision models are compared and some advices about inventory cash flow management are given. Further, sensitivity analysis is conducted to give more general situations to illustrate the rationality of the management advices.

1. Introduction

In an inventory management, a major concern problem is to decide the time when a replenishment order is to be placed and further what quantity of such replenishment is to be ordered. As is well known continuous review inventory model is an appropriate mathematical model for such problem (Darwish, 2008; Dutta, Chakraborty, & Roy, 2007a). In traditional continuous review inventory systems, the lead-time and the annual demand is assessed by a crisp value (Salameh, Abbound, El-kassar, & Ghattas, 2003; Zhang, Patuwo, & Chu, 2003). But in practical situations, precise values of the cost characteristics are not always known exactly as they may be vague and imprecise.

Fuzzy theory is one of good choices to deal with this situation and many scholars have already made some achievements. Gen, Tsujimura, and Zheng (1997) presented mean value of fuzzy number for continuous review inventory model with fuzzy input data. Yao and Chiang (2003) considered the total cost of inventory without backorder and compared the results obtained by two defuzzification methods. Dutta, Chakraborty, and Roy (2007b) analyzed a single period inventory model with fuzzy random demand, and the objective was to determine the optimal order quantity in maximizing the expected profit. Tutuncu, Akoz, Apaydin, and Petrovic (2008) presented new models of continuous review inventory control with or without models in the presence of uncertainty, and defined fuzzy total annual cost functions involving fuzzy arithmetic operations. Recently, Handfield, Warsing, and Wu (2009) developed a (Q, r) model using fuzzy set representations of various uncertain sources including demand, lead time and penalty cost, then the total cost is computed using defuzzification methods.

Moreover, continuous review inventory models should be studied with a mixture of backorder and lost sales. Obviously, when dealing with continuous review inventory problems, most of researchers prefer to assume that the demand during the stock-out period is either completely backorder or completely lost. However, these are always not true. It can be observed, in real market, that when the inventory system is out of stock, a fraction of the customers are willing to fill their demand immediately from another source, while others may wait till the next arrival of stock. Hence some continuous review inventory models were extended to include backorder and lost sales. Vijayan and Kumaran (2008) considered continuous review inventory models in which a fraction of demand was backordered and the remaining fraction was lost during the stock-out period under fuzzy environment. They assumed fixed ordering cost, inventory holding cost, fixed shortage cost and shortage cost of lost sales are fuzzy. In fact, compared with demand, they are easier to obtain accurate data for companies. In the present day scenario, it is really very difficult to forecast the demand for a decision make. On this view, fuzzy inventory models in situations where the customer demand is described imprecisely should be studied in detail.

In this paper, we consider a continuous review inventory model under fuzzy environment by assuming average annual demand

* Corresponding author.
E-mail address: wanglin982@163.com (L. Wang).

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and demand during lead-time as fuzzy variables. And during the stock-out period a part of demand is backordered and the remaining fraction is lost. Then an elegant methodology to determine the optimal order quantity and reorder point that the expected total cost per year has a minimum value. Sometimes, decision makers are not concern about making the total cost minimum, but hope that the total cost does not exceed the budget level, especially when enterprises have much cash flow pressure. In this situation, a fuzzy dependent-chance programming (DCP) model is always regarded as a good choice. The DCP model can be used to find the optimal solution for maximizing the credibility of an event that the total cost in the planning periods does not exceed a certain budget level. Liu (2000) provided the framework of DCP and illustrated a fuzzy simulation (FS) approach for measuring possibility using genetic algorithm by some numerical examples. Wang, Tang, and Zhao (2007) constructed a fuzzy DCP EOQ model in which the function was about credibility and find that it is very difficult to obtain the value with an analytic method. So, they designed a FS approach to estimate credibility of DCP model for fixed Q and r.

When the objective functions to be optimized are multimodal or the search spaces are particularly irregular, hybrid intelligent algorithms using FS and optimization algorithms should be designed to solve the fuzzy models. Moreover, optimization algorithms need to be highly robust in order to avoid getting stuck at local optimal solution. Among these algorithms, genetic algorithm (GA) has been proved to be effective for the FS approach (Ke & Liu, 2010; Liu, 2000). However, the GA is only capable of identifying the high performance region at an affordable time and displays inherent difficulties in performing local search for numerical applications. So, it is necessary to find a novel algorithm to deal with FS more efficiently and effectively.

Recently, a novel differential evolution (DE) algorithm was proposed by Storn and Price (1997) for complex continuous non-linear, on-differentiable and multi-modal optimization problem. This technique combines simple arithmetic operators with the classical events of crossover, mutation and selection to evolve from a randomly generated starting population to a final solution. DE algorithm is easy to implement, requires only several parameters and shows fast convergence (Aslantas & Kurban, 2010; Chang, 2010; Wang, He, & Zeng, 2011). It is reliable, accurate, robust and fast optimization that make DE algorithm widely used (Lu, Zhou, Qin, Li, & Zhang, 2010; Wang, He, Wu, & Zeng, 2011). However, according to Krink, Filipic, and Fogel (2004), noise may adversely affect the performance of DE due to its greedy nature. How to improve the performance of DE algorithm is a hot focus. There are two methods to obtain better behaviors than the typical DE algorithm. One is improving the parameters of DE, and the other introduces mechanisms of other algorithms into DE. So in this paper, in order to search the optimum solution, two hybrid DE algorithms are also used besides the basic DE algorithms. One called self-adaptive DE (SDE) is an attempt to dynamically adjust F, a scale factor used to control the amplification of the differential variation. SDE is proposed by Salman, Engelbrech, and Omran (2007), and the performance of SDE is investigated and compared with other well-known approaches. The experiments conducted show that SDE generally outperform DE algorithm in all the benchmark. The other one called modified DE (MDE) is in the framework of DE owning new mutation operator and selection mechanism inspired from GA, particle swarm optimization (PSO) and simulated annealing (SA), respectively. In other words, positive characteristics of DE, GA, PSO and SA are combined to create a new efficient stochastic search technique. The algorithm has been successfully applied to solve the non-convex economic dispatch (Amjady & Sharifzadeh, 2010). It is examined on three economic dispatch test systems and compared with some the most recently published economic dispatch solution methods to show the efficiency and robustness of SDE. So, three novel approaches using FS and DE/SDE/MDE are designed to solve DCP model, named FSDE/FSSDE/FSMDE respectively.

The aim of this paper is to find the optimal decision for continuous review inventory models with a mixture of backorders and lost sales under fuzzy demand. So, we propose two decision methods under different situations. The first one is minimizing the expected total cost per year, and the second one is maximizing the credibility of an event such that the total cost in the planning periods does not exceed a certain budget level. Then we will compare the two methods and try to provide some advices about the principals of inventory cash flow management. This paper will also present a DCP model about continuous review inventory system under the practical environment for the first time. Moreover, new hybrid intelligent algorithms will be proposed to solve the DCP model in an efficient and reliable way without considering the shapes of fuzzy membership functions.

The rest of this paper is organized as follows: Section 2 is preliminaries. Section 3 contains the proposed models and analysis. In Section 4, we defuzzify the fuzzy model by possibilistic mean value to find the optimum solution. In Section 5, we propose three hybrid intelligent DE algorithms to solve the DCP model. Section 6 contains numerical examples, results and advises. In Section 7, we discuss the result and provide directions for future research.

2. Fuzzy preliminaries

Definition 1. A triangular fuzzy number $\widetilde{A} = (\underline{A}, A, \overline{A})$ on the space of real numbers R, can be described with following membership function:

$$\mu_{\widetilde{A}}(x) = \begin{cases} L(x) = (x - \underline{A})/(A - \underline{A}) & \text{for } \underline{A} \leqslant x \leqslant A \\ R(x) = (\overline{A} - x)/(\overline{A} - A) & \text{for } \overline{A} \leqslant x \leqslant A \end{cases} \tag{1}$$

For a given fuzzy set $\widetilde{A}$, the alpha-cut set A_α is given by $A_\alpha = \{x | \mu_{\widetilde{A}}(x) \geqslant \alpha\}$ and is denoted by interval $[A_\alpha^-, A_\alpha^+], 0 \leqslant \alpha \leqslant 1$.

Definition 2. For a given fuzzy number $\widetilde{A}$, the interval value possibilistic mean is defined as $M(\widetilde{A}) = [M_*(\widetilde{A}), M^*(\widetilde{A})]$ where $M_*(\widetilde{A})$ and $M^*(\widetilde{A})$ are the lower and upper possibilistic mean values of $\widetilde{A}$ (Carlsson & Fullér, 2001) and are defined respectively by

$$M_*(\widetilde{A}) = \frac{\int_0^1 \alpha A_\alpha^- \mathrm{d}\alpha}{\int_0^1 \alpha \mathrm{d}\alpha}, \quad M^*(\widetilde{A}) = \frac{\int_0^1 \alpha A_\alpha^+ \mathrm{d}\alpha}{\int_0^1 \alpha \mathrm{d}\alpha} \tag{2}$$

The possibilistic mean value of $\widetilde{A}$ is defined as

$$\overline{M}(\widetilde{A}) = \frac{M_*(\widetilde{A}) + M^*(\widetilde{A})}{2} \tag{3}$$

In other words, one can write

$$\overline{M}(\widetilde{A}) = \int_0^1 \alpha(A_\alpha^- + A_\alpha^+)\mathrm{d}\alpha \tag{4}$$

Definition 3. In applications, let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers with $A_\alpha = [A_\alpha^-, A_\alpha^+]$ and $B_\alpha = [B_\alpha^-, B_\alpha^+], \alpha \in [0, 1]$ then for ranking fuzzy numbers, $\widetilde{A} \leqslant \widetilde{B} \Longleftrightarrow \overline{M}(\widetilde{A}) \leqslant \overline{M}(\widetilde{B})$.

Let Θ be a nonempty set, $\mathrm{P}(\Theta)$ be the power set of Θ, and Pos be a possibility measure. Then the triplet $(\Theta, \mathrm{P}(\Theta), \mathrm{Pos})$ is called a possibility space. Let A be a set in $\mathrm{P}(\Theta)$. The necessity measure of A can then represented by

$$\mathrm{Nec}\{A\} = 1 - \mathrm{Pos}\{A^C\} \tag{5}$$