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Inventory ordering policies of delayed deteriorating items under permissible delay in payments

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ABSTRACT

The existing literature on inventory of deteriorating items considers deterioration to begin as soon as the items are stocked. However, there are several deteriorating items that do not start deteriorating immediately they are held in stock. Some farm produce like potatoes, yams and even some fruits and vegetables have this property. Depletion of these items as soon as they are stocked will depend on demand, and when deterioration begins, it will depend on both demand and deterioration. In this paper, we develop a mathematical model on the inventory of deteriorating items that do not start deteriorating immediately they are stocked. The model also takes into cognizance the fact that in business activities nowadays customers are given some allowed period within which to settle for the goods supplied to them. They can use the accrued money from sales of the supplied goods to earn interest within the allowed period. They are charged interest only when they fail to settle the amount they owe the supplier at the end of the allowed period.

1. Introduction

In general cases, inventory depletion is considered to be as a result of demand rate. However, in real life situations, there are cases of inventory depletion due to deterioration or other factors. The decaying inventory problem was first analyzed by Ghare and Shrader (1963) who developed EOQ model with a constant rate of decay. This work was extended by Covert and Philip (1973) who developed an EOQ model for a variable rate of deterioration. Shah (1977) generalized the work of Ghare and Shrader to allow for backordering. Hollier and Mark (1983) developed a model for inventory replenishment policies for deteriorating items in a declining market. Other authors that worked on deteriorating items without considering delay in payments include Heng et al. (1991), Datta and Pal (1990) and Goswami and Chaudari (1991) who considered demand to be linear. Others are Papachristos and Skouri (2000) and Chang and Dye (1999) who looked at deteriorating items in a continuous review environment with shortages, while Cheng and Chen (2004) considered deteriorating items in a periodic review environment with shortages.

Condition of permissible delay in payment was not considered prior to 1985 when Goyal (1985) in a maiden paper developed an EOQ model under such condition. Aggarwal and Jaggi (1995) extended Goyal's model for deteriorating items. Jamal et al. (1997) extended the work of Aggarwal and Jaggi by allowing for shortages. Other authors that extended the work of Goyal include Meddah et al. (2004) who investigated the effect of permissible delay in a periodic review environment, Ouyang et al. (2005) who added the condition of cash discount and Teng (2002) who considered the difference between unit price and unit cost of items. Others are Shin et al. (1996), Jamal et al. (2000) and Liao et al. (2000) who looked at delay in payments under inflation. Permissible delay in payments under continuous review environment was looked at by Salameh et al. (2003) while Chung and Huang (2003)) looked at the situation under the EPQ production model. Huang (2003), Chung et al. (2005) and Teng (2002) all developed models under permissible delay in payments but where the condition is dependent on some other conditions such as the ordering quantity, retailer/wholesaler relationship or conditional permissible delay in payments.

In our case we extend Goyal's work (Goyal, 1985) to a situation of delayed deteriorating items, i.e. items that do not deteriorate immediately they are stocked. During this period, before deterioration sets in, depletion of inventory is dependent only on demand. As deterioration sets in, depletion is then dependent on both demand and deterioration. Several items in tropical countries exhibit this property. These items include farm produce such as potatoes, yams, some fruits and vegetables, etc. The demand rate in this case remains constant from the time they are stocked, up to the time the deterioration sets in. Depletion of inventory from the time the deterioration begins

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up to the end of the cycle will depend on both deterioration and demand.

In business transaction nowadays, it is commonly known that customers are allowed some period of time within which to settle for the goods supplied to them. This is an advantage to the customer, who is not expected to pay for the goods supplied immediately after delivery; instead, payment can be delayed up to the end of the allowed period. The customer does not pay interest during the fixed period given to settle for the goods supplied, but if payment is delayed beyond this period, interest is charged. Our model takes cognizance of this trend.

During the refereeing process of this paper, our attention was drawn to some recent works by Ouyang et al. (2006) and Chung (2009) who developed an inventory model with quite similar assumptions. In their model the demand rate before and after the deterioration sets in is a constant. Moreover the product life t has a probability density function $f(t)=\theta_1 e^{-\theta_1(t-t_1)}$ for $t>T_1$, so the deterioration rate is given by $r(t)=f(t)/1-F(t)$ for $t>T_1$ where $F(t)=\int_{T_1}^{t} f(x)dx=1-e^{-\theta_1(t-T_1)}$ for $t>T_1$ and θ_1 is a parameter.

In our case however, the demand rate before deterioration sets in is different from the rate after the start of deterioration. This is a reasonable assumption since when deterioration sets in, the demand for the items will change, which is one of the major contributions of this paper to the existing literature. Moreover our model has a constant deterioration rate, as it is in the papers of Aggarwal and Jaggi (1995), Jamal et al. (1997), Ouyang et al. (2005), Liao et al. (2000) and Chang and Dye (1999). In our original paper, the way we calculated the interest earned, was also different from the way they did theirs. However, we decided to change the way we calculated interest earned to be like their own so as to make our model a generalization of their own. In this way, if $D_1=D_2$ then our model reduces to their own. We also tried as much as possible to follow their pattern of presentation for this purpose.

In our analysis we implore the more widely used average annual cost method rather than the NPV (discounted cash flow) method. The NPV approach is generally considered as the right approach for studying inventory control systems and this is due to some cogent reasons, see for instance (Grubbstrom, 1980; van der Laan, 2003). There are situations in particular, such as in reverse logistics inventory models, where the average annual cost approach could be inappropriate to use (Teunter and van der Laan, 2002). The NPV approach would certainly be very helpful in analyzing inventory models dealing with permissible delay in payments. This is because the approach would provide a framework for an in-depth analysis on what the customer earns over time, as a result of the grace period given by the supplier. It would also show clearly what the customer loses over time, due to interest given to the supplier, if the customer defaults in payment at the due date. It is worth noting however, that the average annual cost method is a good approximation to the NPV method for EOQ models such as ours. This is in particular true, for parameter values that might reasonably be encountered in practice (Hadley, 1964) and for non-slow-moving items (Klein Haneveld and Teunter, 1998).

2. The mathematical model

In developing the model, we use the following notation and assumptions:

2.1. Notation

D_1	demand rate (units per unit time) during the period before the deterioration sets in
D_2	demand rate (units per unit time) after the deterioration sets in
Q	order quantity (units per order)
T	inventory cycle length (time units)
T_1	time the deterioration sets in
T_2	difference between the inventory cycle length and the time the deterioration sets in
I_d	inventory level at the time the deterioration begins
C	unit cost of the item (in Naira or Euro say)
S	unit selling price of the item (in Naira)
A	ordering cost per order (Naira per order)
i	inventory carrying charge(excluding interest charges, per Naira per unit time)
P	interest payable per cycle
I_p	interest paid per Naira investment in stocks per year
I_e	interest that can be earned per Naira investment in a year
M	permissible delay in settlement of the account
θ	rate of deterioration
E_1	interest earned from the accrued sales
$d(T_2)$	number of items that deteriorate during the time interval, $[T_1,T_2]$

2.2. Assumptions

(i) Replenishment is instantaneous
(ii) Lead time is negligible
(iii) Shortages are not allowed

2.3. Model development

Let $I(t)$ be the inventory level at any time t, $(0\le t\le T)$ (Figs. 1 and 2). Depletion of inventory from the beginning of the cycle up to the time the deterioration sets in will occur only due to demand. When deterioration sets in, depletion will then occur due to both demand and deterioration. The differential equations which describe the variation of the inventory level during $[0, T]$ are

$$\frac{dI(t)}{dt}=-D_1, \quad 0\le t\le T_1 \tag{1}$$

with boundary condition $I(T_1^-)=I(T_1^+)$ and

$$\frac{dI(t)}{dt}=-D_2-\theta I(t), \quad T_1\le t\le T \tag{2}$$

with boundary condition $I(T)=0$

Their solutions are

$$I(t)=D_1(T_1-t)+\frac{D_2}{\theta}(e^{\theta(T-T_1)}-1), \quad 0\le t\le T_1 \tag{3}$$

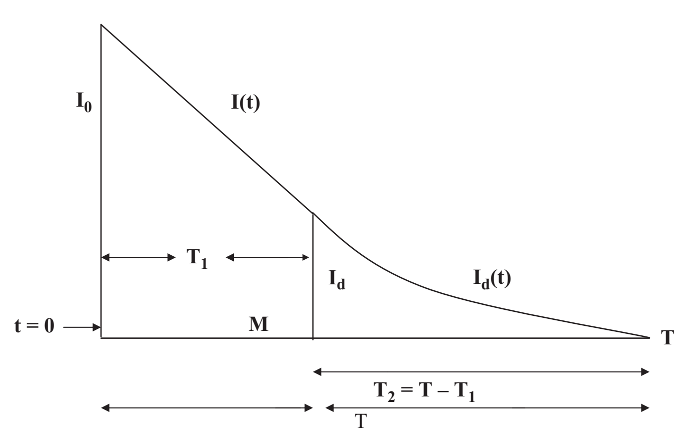

Fig. 1. Inventory position in a review period, T for $0\le M\le T_1<T$.