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A serial mixed produce-to-order and produce-in-advance inventory model with multiple retailers

Yunan Wu [a,*], T.C.E. Cheng [b], Jihong Zhang [a]

[a] School of International Business, Beijing Foreign Studies University, Beijing 100089, China
[b] Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

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ABSTRACT

We formulate a general mixed produce-to-order and produce-in-advance inventory model having multiple stocking echelons and multiple retailers. We show that the problem to find an optimal inventory policy for such a model with a uniform or a normal demand distribution can be reduced to a general constrained optimization problem.

1. Introduction

The major challenge facing producers of perishable or decaying products is to plan for the production and inventory levels of their products to meet random demands over the sale cycle dictated by the products' limited life. Generally, such products are made by a processing system consisting of a series of processing stages. We assume that the system comprises $n-1$ serial processing facilities, so it includes n stocking echelons, which hold various kinds of inventory items in the form of raw materials (echelon 1), different grades of work-in-process (echelons 2 to $n-1$) and the finished product (echelon n). The last echelon n is the retailer. Since there are usually more than one retailer for the product, echelon n contains $m \geq 1$ retailers. A diagrammatic representation of such a processing system is given in Fig. 1.

Because of the limited product life, there are no opportunities for the producer to reorder more raw materials from the supplier, neither is there time for the producer to modify the initial inventory levels of the different grades of work-in-process at the stocking echelons. Since the demand for the product is random and the product has a limited life, if the producer overproduces, he will have to liquidate all excess inventory held at the stocking echelons at the end of the sale cycle. This will reduce his profit over the sale cycle. However, the liquidation costs will be lower for work-in-process inventory items held at lower stocking echelons as they have been added less value. On the other hand, if the producer under-produces, he can convert some of the work-in-process inventory into the final product to meet the excess demand within the sale cycle. However, the profit over the sale cycle is lower again in this case because extra costs will be incurred to expedite the orders, and a higher production cost will result from converting the partially decayed work-in-process inventory into the final product (e.g., more energy and additives are needed). Therefore, the optimal inventory level at each echelon needs to be determined at the start of each sale cycle with the objective of maximizing the expected profit over the sale cycle.

Practical examples of such a model is the production of oriental health and herbal food, such as readily served swallow nest soup. The raw material is typically a protein-based solution, which has a short life of 5–7 days that defines the duration of the sale cycle. Ingredients such as herbs, vitamins, nutrients, scents, colouring, additives, etc. are added at different processing stages organized in a serial configuration. Each processing stage provides a unique treatment, which takes a short time compared with the cycle duration, and produces a different grade of work-in-process. The grade of the work-in-process increases with the echelon level. So it will be more profitable, and it will take a shorter time, to convert the work-in-process inventory at higher echelons to the final product, if demand for the product exceeds supply within the sale cycle. However, since the work-in-process at higher echelons has a higher added value, it will be more costly to liquidate any such excess inventory at the end of the sale cycle.

Generally, there are two kinds of inventory policies used for production and inventory planning. One is produce-in-advance (PIA) and the other is produce-to-order (PTO). Under PTO, if a shortage of the finished product occurs, the excess demand can be partially satisfied by converting some of the available raw materials and work-in-process into products. Since customers are usually unwilling to wait for a long time, the fraction of the work-in-process converted to meet the excess demand is smaller

* Corresponding author.
E-mail address: wuyunan@bfsu.edu.cn (Yunan Wu).

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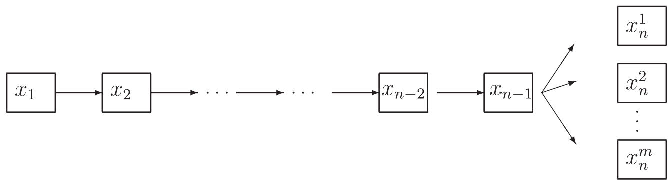

Fig. 1. A serial production–inventory system comprising n stocking echelons and m retailers.

than the unsatisfied demand that is carried over to a lower echelon. Thus, adopting the PTO inventory policy will result in a loss of the customers who are unwilling to wait. Moreover, as far as production is concerned, the unit cost of a finished product under PTO is higher than that under PIA, because for the latter there is no need for rush orders, and production can be planned and implemented well in advance. On the other hand, if the PIA inventory policy is used, liquidation loss will be incurred if supply exceeds demand because of the wasted processing costs and the disposal of unused inventory. Therefore, as observed by Eynan and Rosenblatt (1995), a composite of PIA and PTO is the dominating policy.

Whitin (1953) was the first to present such a single-period problem with stochastic demand and two echelons. Hariga (1998) considered a model composed of n echelons. Subsequently, many researchers have generalized his model to different forms (see, for example, Bivin, 2003; Wong et al., 2009; Banerjee, 2005).

In Section 2 we introduce the notation used throughout the paper and study a serial mixed PTO and PIA inventory model comprising n echelons and m retailers. We formulate the general composite model under two kinds of demand distribution, namely uniformly distributed demands and normally distributed demands, and show that the problem can be reduced to a general constrained optimization problem in Section 3. Finally, we conclude the paper in Section 4.

2. A serial mixed produce-to-order and produce-in-advance inventory model

As shown in Fig. 1, we consider a single-period stochastic production–inventory model composed of n stocking echelons, which hold various inventory items in the form of raw materials (echelon 1), different grades of work-in-process (echelons 2 to $n-1$) and the finished product (echelon n). For echelon n, we assume that it consists of m retailers.

We use the following notation:

$T=\{1,2,\ldots,m\}$, the set of retailers;

$S=\{1,2,\ldots,n\}$, the set of echelons in the production–inventory system;

$D^j=$ the random demand of the jth retailer, $j\in T$;

$x_i=$ inventory at the ith echelon (a decision variable), $i=1,2,\ldots,n-1$;

$x_n^j=$ inventory of the jth retailer at echelon n (a decision variable), $j\in T$;

$x=(x_1,x_2,\ldots,x_{n-1},x_n^1,x_n^2,\ldots,x_n^m)$ is an $(n+m-1)$-dimensional decision vector;

$p_n=$ unit profit (excluding liquidation loss) when the finished products are produced in advance. It is the difference between the unit selling price and the unit total cost, which consists of the setup, production, inventory holding and transportation costs at all of the echelons;

$p_i=$ unit profit (excluding liquidation loss) when the finished products are produced to order by converting the work-in-process inventory at echelon i, $i=1,2,\ldots,n-1$. Obviously, $p_1\le p_2\le\cdots\le p_{n-1}\le p_n$ as explained in Section 1;

$l_i=$ unit liquidation loss for unused inventory at the ith echelon. We know that $l_1\le l_2\le\cdots\le l_{n-1}\le l_n$ as discussed in Section 1;

$u_i=$ the fraction of excess demand that may be satisfied by producing the finished products from inventory items held at echelon i. From Section 1, we know that $u_1\le u_2\le\cdots\le u_{n-1}\le u_n=1$.

Next we formulate the single-period stochastic production–inventory model with n stocking echelons and m retailers at echelon n under the composite policy of produce-to-order and produce-in-advance. We consider the model in which a retailer can obtain the finished products first from another retailer, then from echelon $n-1$, if the demand cannot be satisfied from its initial stock. Since retailers can transfer their products to one another, we only need to consider the sum of demands occurring at all retailers. Let $q=$unit cost of transferring the finished product from one retailer to another, where q consists of the transportation costs, storage fees, etc.

There are $n+1$ possible scenarios to consider as follows.

Scenario $n+1$: $0\le\sum_{j=1}^m D^j\le\sum_{j=1}^m x_n^j$. That is, for all retailers, the sum of all random demands does not exceed the sum of their inventory, and the finished products at echelon n that have been produced in advance satisfy the demands. So there is no need to convert raw materials or work-in-process at any lower echelons. Thus, we need to consider the liquidation loss of the raw materials at echelon 1, work-in-process at echelons 2 to $n-1$ and excess finished products, i.e., $\sum_{j=1}^m x_n^j-\sum_{j=1}^m D^j$, at echelon n. However, for some retailers, it is possible that their demand cannot be met by their own stocks and the finished products need to be transferred from other retailers. So the transferring cost needs to be considered. Set $A_{n+1}=\sum_{j=1}^m \chi_{\{D^j>x_n^j\}}(D^j-x_n^j)$, where $\chi_E(\omega)=1$ if $\omega\in E$, otherwise $\chi_E(\omega)=0$, and the profit will be

$$P_{n+1}(x)=p_n\left(\sum_{j=1}^m D^j\right)-l_n\left(\sum_{j=1}^m x_n^j-\sum_{j=1}^m D^j\right)-\sum_{i=1}^{n-1} l_i x_i-qA_{n+1}.$$

Scenario n: $\sum_{j=1}^m x_n^j\le\sum_{j=1}^m D^j$ and $u_{n-1}(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j)\le x_{n-1}$. That is, $\sum_{j=1}^m x_n^j\le\sum_{j=1}^m D^j\le\sum_{j=1}^m x_n^j+(x_{n-1}/u_{n-1})$. We know that all initial stocks at echelon n cannot satisfy all demands and the excess demand can be met by converting the work-in-process at echelon $n-1$. Moreover, the amounts x_i, $i=1,2,\ldots,n-2$, and $x_{n-1}-u_{n-1}(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j)$ will remain at echelons 1 to $n-1$, respectively. Set $A=\sum_{j=1}^m \chi_{\{0\le D^j\le x_n^j\}}(x_n^j-D^j)$, and the transferring cost is qA. Thus, the profit will be

$$P_n(x)=p_n\left(\sum_{j=1}^m x_n^j\right)+p_{n-1}u_{n-1}\left(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j\right)$$
$$-l_{n-1}\left[x_{n-1}-u_{n-1}\left(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j\right)\right]-\sum_{i=1}^{n-2} l_i x_i-qA.$$

Scenario $n-1$: $u_{n-1}(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j)\ge x_{n-1}$ and $u_{n-2}(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j-(x_{n-1}/u_{n-1}))\le x_{n-2}$. That is, $\sum_{j=1}^m x_n^j+(x_{n-1}/u_{n-1})\le\sum_{j=1}^m D^j\le\sum_{j=1}^m x_n^j+(x_{n-1}/u_{n-1})+(x_{n-2}/u_{n-2})$. In this case, the finished products at echelon n, all work-in-process at echelon $n-1$ and a part of the work-in-process, i.e., $u_{n-2}(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j-(x_{n-1}/u_{n-1}))$, at echelon $n-2$ will be exhausted. All quantities initially available at echelons 1 to $n-3$ and a part of the stock at echelon $n-2$, i.e., $x_{n-2}-u_{n-2}$ $(\sum_{j=1}^m D^j-\sum_{j=1}^m x_n^j-(x_{n-1}/u_{n-1}))$, remain. We also consider the transferring cost at echelon n.