Int. J. Production Economics 136 (2012) 384–388

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An improved algorithm and solution on an integrated production-inventory model in a three-layer supply chain

Leopoldo Eduardo Cárdenas-Barrón [a,b,*], Jinn-Tsair Teng [c], Gerardo Treviño-Garza [b], Hui-Ming Wee [d], Kuo-Ren Lou [e]

[a] Department of Industrial and Systems Engineering, School of Engineering, InstitutoTecnológico y de Estudios Superiores de Monterrey, E. Garza Sada 2501 Sur, CP 64 849, Monterrey, Nuevo León, México
[b] Department of Marketing and International Business, School of Business, Instituto Tecnológico y de Estudios Superiores de Monterrey, E. Garza Sada 2501 Sur, CP 64 849, Monterrey, Nuevo León, México
[c] Department of Marketing and Management Sciences, The William Paterson University of New Jersey, Wayne, NJ 074702103, USA
[d] Department of Industrial and Systems Engineering, Chung Yuan Christian University, Chungli 32023, Taiwan, ROC
[e] Department of Management Sciences and Decision Making Tamkang University, Tamsui, Taipei 251, ROC

ARTICLE INFO

Article history:
Received 9 December 2010
Accepted 7 December 2011
Available online 16 December 2011

Keywords:
Production planning
Inventory control
Supply chain coordination
Joint economic lot sizing

ABSTRACT

Ben-Daya et al. (2010) established a joint economic lot-sizing problem (JELP) for a three-layer supply chain with one supplier, one manufacturer, and multiple retailers, and then proposed a heuristic algorithm to obtain the integral values of four discrete variables in the JELP. In this paper, we first complement some shortcomings in Ben-Daya et al. (2010), and then propose a simpler improved alternative algorithm to obtain the four integral decision variables. The proposed algorithm provides not only less CPU time but also less total cost to operate than the algorithm by Ben-Daya et al. (2010). Furthermore, our proposed algorithm can solve certain problems, which cannot be solved by theirs. Finally, the solution obtained by the proposed algorithm is indeed a global optimal solution in each of all instances tested.

1. Introduction

To bear a better resemblance to practice, Ben-Daya et al. (2010) considered a joint economic lot-sizing problem (JELP) in a three-layer supply chain with one supplier, one manufacturer, and multiple retailers as follows: The retailers have a common basic cycle time T. The manufacturer has the cycle time $T_m = K_2T$ while the supplier has the cycle time $T_s = K_1T_m = K_1(K_2T)$. The supplier receives m_1 equal shipments of raw materials during its cycle time T_s, transforms them into semi-finished products, and delivers m_2 equal-sized batches to the manufacturer during the manufacturer's cycle time T_m. The manufacturer, in turn, transforms those semi-finished products into finished products and ships finished products to each retailer at its order quantity every T units of time. However, the order quantity received by a retailer might be different from those received by the others. Then they established the chain-wide annual total cost (a.k.a., the total cost) as a function of K_1, K_2, T, m_1, and m_2 using the sum of the costs incurred by the supplier, the manufacturer, and the retailers. They minimized the chain-wide annual total cost in which four variables (i.e., K_1, K_2, m_1, and m_2) are discrete positive integers, and the other T is a real number. Notice that their JELP is a nonlinear integer programming (NLIP) model, and thus is hard to find an optimal solution using an exact method. Furthermore, the JELP is complex and computationally intensive even using mathematical software such as LINGO to solve it. By relaxing all integral variables as continuous variables, Ben-Daya et al. (2010) derived a near optimal solution to the problem using an algebraic method of completing perfect square without classical differential calculus techniques. In general, most studies use the classical differential calculus method to obtain the optimal values of the continuous decision variables. However, an algebraic method of perfect squares has been used in optimization problems in the inventory field recently. Examples are Grubbström (1995), Grubbström and Erdem (1999), Cárdenas-Barrón (2001, 2007, 2008), and Sphicas (2006), just to name a few. For an up-to-date review on different optimization approaches in inventory lot-sizing problems, see Cárdenas-Barrón (2011).

Ben-Daya et al. (2010) solved the relaxed JELP (i.e., relaxing all discrete integral variables in JELP as continuous real-number

* Corresponding author at: Department of Industrial and Systems Engineering, School of Engineering, Instituto Tecnológico y de Estudios Superiores de Monterrey, E. Garza Sada 2501 Sur, CP 64 849, Monterrey, Nuevo León, México. Tel.:+52 81 83 28 42 35; fax: +52 81 83 28 41 53.
E-mail addresses: lecarden@itesm.mx (L.E. Cárdenas-Barrón), TengJ@wpunj.edu (J.-T. Teng), gerardo.trevino@itesm.mx (G. Treviño-Garza), weehm@cycu.edu.tw (H.-M. Wee), 109880@mail.tku.edu.tw (K.-R. Lou).

0925-5273/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2011.12.013

variables) by an algebraic method of completing the square, then proposed an algorithm to find the integral values for those four discrete integral variables. However, their proposed integral procedure seems to be computationally expensive. In fact, their algorithm requires to compute the integral variables (K_1, K_2), the continuous variable (T), and the total cost function for several times. For simplicity, we set $\lceil w \rceil$ as the smallest integer which is greater than or equal to w. Then their algorithm requires evaluating the values of K_1, K_2, T, and the total cost TC for $4\lceil m_1 \rceil \lceil m_2 \rceil$ times, if both $\lceil m_1 \rceil$ and $\lceil m_2 \rceil$ are greater than one. If any of $\lceil m_1 \rceil$ or $\lceil m_2 \rceil$ is equal to 1, then the number of evaluations is less than or equal to $4\lceil m_1 \rceil \lceil m_2 \rceil$. For example, if $\lceil m_1 \rceil = 11$ and $\lceil m_2 \rceil = 13$, then their algorithm requires to compute each of K_1, K_2, T, and the total cost TC for 572 times.

In this paper, we first complement mathematical errors in Ben-Daya et al. (2010) on the optimal basic cycle time $T^* = \sqrt{W/Y}$ and the minimum value for the annual total cost $TC = 2\sqrt{WY}$. If α_2 in Ben-Daya et al. (2010) is negative then both $K_2 = \sqrt{\alpha_2 \phi_2 / (\psi_2 \sum O_r)}$ in (29) and $Y = (K_2\phi_2 + \alpha_2)/2$ are not real numbers. Consequently, neither optimal basic cycle time $T^* = \sqrt{W/Y}$ nor the annual total cost $TC = 2\sqrt{WY}$ is a real number. This contradicts to the facts that both T^* and TC are real numbers. Hence, for correctness and completeness, we need to discuss the case in which $\alpha_2 < 0$. For simplicity, we discuss and illustrate this case using a numerical example as Instance 14 in Section 3 later. We then rearrange the total cost in (27) in Ben-Daya et al. (2010), and then propose a simple integral procedure similar to that by García-Laguna et al. (2010) to obtain the integral values for those four discrete variables m_1, m_2,K_1, and K_2. In addition, the proposed integral procedure discriminates the situations in which there is only one solution and when there are two solutions for each discrete variable. Furthermore, we not only obtain the integral values for all discrete variables in simple-to-apply closed-form expressions, but also need to compute the value of the continuous variable (T) only once, instead of $4\lceil m_1 \rceil \lceil m_2 \rceil$ times using the algorithm in Ben-Daya et al. (2010).

2. Mathematical model and algorithm

For simplicity, we use the same notation and assumptions as in Ben-Daya et al. (2010). After some mathematical manipulations, the annual total cost for the entire supply chain in (27) in Ben-Daya et al. (2010) can be re-written as follows:

$$TC(m_1,m_2,K_1,K_2) = \sqrt{2}\left\{\sqrt{f_1+f_2+f_3+f_4+e}\right\} \tag{1}$$

where f_1, f_2, f_3, f_4, and e are given by

$$f_1 = LO_s m_1 + \frac{GA_s}{m_1},$$

$$f_2 = ZO_m m_2 + \frac{XA_m}{m_2},$$

$$f_3 = \psi_1 (A_m + O_m m_2) K_1 + \frac{\alpha_1 (A_s + O_s m_1)}{K_1},$$

$$f_4 = \psi_2 \left(\sum_{r=1}^{n_r} O_r\right) K_2 + \frac{\alpha_2 \phi_2}{K_2},$$

and

$$e = LA_s + GO_s + ZA_m + XO_m + \alpha_2 \sum_{r=1}^{n_r} O_r.$$

To avoid taking a square root of a negative number as shown in (29) in Ben-Daya et al. (2010), we examine the values of G, L, X, Z, ψ_1, ψ_2, α_1, α_2, and ϕ_2 as follows:

$$G = \frac{h_0 D^2}{P_s} > 0,$$

$$L = h_s D(1 - D/P_s) > 0,$$

$$X = \frac{2h_s D^2}{P_s} > 0,$$

$$Z = \frac{h_s D^2}{P_m} - h_s D - \frac{h_m D^2}{P_m} + h_m D = D\left(1 - \frac{D}{P_m}\right)(h_m - h_s),$$

$$\psi_1 = \frac{G}{m_1} + L > 0,$$

$$\psi_2 = K_1 \psi_1 + \alpha_1,$$

$$\alpha_1 = \frac{X}{m_2} + Z,$$

$$\alpha_2 = \frac{2h_m D^2}{P_m} - h_m D + \sum_{r=1}^{n_r} h_r D_r = h_m D\left(\frac{2D - P_m}{P_m}\right) + \sum_{r=1}^{n_r} h_r D_r,$$

and

$$\phi_2 = \frac{A_s + O_s m_1}{K_1} + A_m + O_m m_2 > 0.$$

To minimize (1) is equivalent to minimize $\sum_{i=1}^{4} f_i$. It is clear that each f_i, $i=1$, 2, 3, and 4, has the similar mathematical form as $a_1 y + a_2/y$. For the following cost-minimizing problem:

Minimizing $a_1 y + a_2/y$ when both a_1 and a_2 are positive, and y is a positive integral decision variable,

García-Laguna et al. (2010) proved that the optimal integral solution is as follows:

$$y = \left\lceil -0.5 + \sqrt{0.25 + \frac{a_2}{a_1}} \right\rceil \text{ or}$$

$$y = \left\lfloor 0.5 + \sqrt{0.25 + \frac{a_2}{a_1}} \right\rfloor, \tag{2}$$

where $\lceil w \rceil$ and $\lfloor w \rfloor$ are the smallest integer greater than or equal to w, and the largest integer less than or equal to w, respectively. Furthermore, it is clear that $\lceil w \rceil = \lfloor w+1 \rfloor$ if and only if w is not an integral value. For this case the problem has a unique optimal solution for y, which is given by anyone of those two mathematical expressions in (2). Otherwise, the problem has two optimal solutions for y: both $y^* = y$ and $y^* = y+1$. This procedure is easy to understand and simple to apply.

In order to apply the closed-form solution as shown in (2) to each discrete variable m_1, m_2,K_1, and K_2, we discuss the corresponding coefficients a_1 and a_2 to each of m_1, m_2,K_1, and K_2 separately as follows:

For m_1, both $G$and L are positive, which imply that both GA_s and LO_s are positive too. Thus, we have

$$m_1 = \left\lceil -0.5 + \sqrt{0.25 + \frac{GA_s}{LO_s}} \right\rceil \text{ or}$$

$$m_1 = \left\lfloor 0.5 + \sqrt{0.25 + \frac{GA_s}{LO_s}} \right\rfloor \tag{3}$$

For m_2, if h_m is greater than h_s, then both Z and ZO_m are positive. Since X is positive, we know that XA_m is positive too.