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Two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation

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ABSTRACT

In many inventory systems, the effects of deterioration of physical goods have drawn much attention of various researchers in recent years. The more the deterioration is, the more the order quantity would be. According to such consideration, taking the deterioration rate into account is necessary. Thus, in this paper, I develop the two-warehouse partial backlogging inventory model introduced in Yang (2006) to incorporate three-parameter Weibull deterioration distribution. The objective is to derive the optimal replenishment policy that minimizes the net present value of total relevant cost per unit time. Two alternative models are compared based on the minimum cost approach. The study shows that the optimal solution not only exists but also is unique. Model 2 is still less expensive to operate than Model 1 in the proposed model. Finally, some numerical examples and sensitivity analysis on parameters are made to validate the results of the proposed inventory system.

1. Introduction

In general, a certain product deteriorates with time varying during the normal storage period, such as seasonal food, vegetables, fruit and others. Thus, to control and maintain the inventory of deteriorating items to satisfy customer's demand or retailer's order is important. In the past few decades, the analysis of deteriorating inventory had been attracted by many researchers' attention. Ghare and Shrader (1963) were the first to propose the inventory model where the factor of deterioration considered was significantly affected. Later, there are many papers presented in the analysis of deteriorating inventory, such as Dave and Patel (1981), Hariga (1996), Teng et al. (1999), and others. Goyal and Giri (2001) presented a review of the inventory literature for deteriorating items since the early 1990s. Recently, there still has many articles explored for the deterioration inventory model by taking different affecting factors into account, such as quantity discount, pricing, partial backlogging, etc.

In practical, the deterioration rate for some items, such as steel, glassware hardware and toys, is so low that there is almost no need to consider deterioration in determining the inventory lot-size. However, some items deteriorated with age, such as the leakage failure of batteries, life expectancy of drugs, etc., the longer the items remained unused, the higher the rate at which they failed. Covert and Philip (1973) developed an inventory model for deteriorating items with variable rate of deterioration. They used the two-parameter Weibull distribution to represent the deterioration distribution. Misra (1975) also used the two-parameter Weibull distribution to fit the deterioration rate for production lot-size model. Wee et al. (2005) proposed a two-warehouse inventory model for deteriorating item with two-parameter Weibull distribution and constant partial backlogging rate under inflation. Lo et al. (2007) proposed an integrated production-inventory model with imperfect production processes under inflation and the product deteriorated with two-parameter Weibull distribution. Recently, Skouri and Konstantaras (2009) considered the order level inventory models for deteriorating seasonable/fashionable products with two-parameter Weibull deterioration rate, time dependent demand and shortages. Concurrently, Skouri et al. (2009) considered an inventory model with general ramp-type demand rate, two-parameter Weibull deterioration rate and partial backlogging following two different replenishment policies.

However, the two-parameter Weibull distribution may not be always applicable in the real life. Some of the items begin to deteriorate only after a period of time in storage, not at the initial. Thus, a three-parameter Weibull distribution is more practical. Philip (1974) generalized the model proposed by Covert and Philip (1973) with three-parameter Weibull distribution. Chakrabarty et al. (1998) extended Philip's model with shortages and linear trend demand. Giri et al. (2003) developed an EOQ model with a three-parameter Weibull deterioration distribution, shortages and ramp-type demand. The major assumptions mentioned in the selected articles are summarized in Table 1.

Note that (1) if the location parameter is zero, then the three-parameter Weibull distribution is reduced to be a two-parameter

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Table 1
Major characteristic of inventory models on selected articles.

Author(s) and published (year)	Demand rate	Single/Two warehouse	Deterioration rate	Allow for shortages	With partial backlogging	Under inflation
Chakrabarty et al. (1998)	Linear demand	Single warehouse	Three-parameter Weibull distribution	Yes	No	No
Covert and Philip (1973)	Constant	Single warehouse	Two-parameter Weibull distribution	No	No	No
Dave and Patel (1981)	Time proportional	Single warehouse	Constant	No	No	No
Ghare and Shrader (1963)	Constant	Single warehouse	Constant	No	No	No
Giri et al. (2003)	Ramp-type demand	Single warehouse	Three-parameter Weibull distribution	Yes	No	No
Hariga (1996)	Time varying (log concave)	Single warehouse	Constant	Yes	No	No
Lo et al. (2007)	Constant	Single warehouse	Two-parameter Weibull distribution	Yes	Yes	Yes
Misra (1975)	Constant	Single warehouse	Two-parameter Weibull distribution	No	No	No
Philip (1974)	Constant	Single warehouse	Three-parameter Weibull distribution	No	No	No
Skouri and Konstantaras (2009)	Time dependent	Single warehouse	Two-parameter Weibull distribution	Yes	Yes	No
Skouri et al. (2009)	Ramp-type	Single warehouse	Two-parameter Weibull distribution	Yes	Yes	No
Teng et al. (1999)	Time varying	Single warehouse	Constant	Yes	No	No
Wee et al. (2005)	Constant	Two warehouses	Two-parameter Weibull distribution	Yes	Yes	Yes
Yang (2006)	Constant	Two warehouses	Constant	Yes	Yes	Yes
Present paper (2010)	Constant	Two warehouses	Three-parameter Weibull distribution	Yes	Yes	Yes

Weibull distribution. (2) If the shape parameter is equal to 1, then the Weibull distribution is reduced to be an exponential form. Thus, the three-parameter Weibull distribution is a generalized distribution to be considered.

Therefore, I here consider the models which introduced in Yang (2006) to develop a two-warehouse partial backlogging inventory model under inflation in which the deterioration rate is a three-parameter Weibull distribution. The deterioration distribution in the two warehouses is independent. Two alternatives are compared based on the minimum cost approach. The study shows that the optimal solution not only exists but also is unique. In general, Model 2 is still less expensive to operate than Model 1 in the proposed model. Finally, some numerical examples for illustration and sensitivity analysis on parameters are provided.

2. Assumptions and notation

The mathematical models of the two-warehouse inventory problems are based on the following assumptions:

1. Lead time is zero and the initial inventory level is zero.
2. The owned warehouse (OW) has a fixed capacity of W units.
3. The rented warehouse (RW) has unlimited capacity.
4. The inventory costs (including holding cost and deterioration cost) in RW are higher than those in OW.
5. Deterioration of the item is a three-parameter Weibull distribution. Deterioration occurs after a period of time as items received into inventory. There is no replacement or repair of deteriorating items during the period under consideration.
6. The deterioration distribution in RW and OW is independent.
7. Each related cost considered is compounded continuously throughout the analysis. Product transactions are followed by instantaneous cash flow.
8. Shortages are allowed. Unsatisfied demand is backlogged, and the fraction of shortages backordered is a differentiable and decreasing function of time t, denoted by $\delta(t)$, where t is the waiting time up to the next replenishment, with $0 \le \delta(t) \le 1$, $\delta(0)=1$ and $\lim_{t\to\infty}\delta(t)=0$. Note that if $\delta(t)=1$ (or 0) for all t, then shortages are completely backlogged (or lost). Moreover, it is assumed that $t\delta(t)$ is an increasing function which satisfies the relation $\delta(t)+t\delta'(t)>0$, where $\delta'(t)$is the derivative of $\delta(t)$.
9. With the viewpoint of cost-minimization, the opportunity cost due to lost sale is the sum of the revenue loss and the cost of lost goodwill. Hence, the opportunity cost due to lost sale here is greater than the unit purchase cost.

In addition, the following notation is used throughout this paper.

$f(t)$	the demand rate at time t, we here assume that $f(t)$ is deterministic at a constant rate of D units per unit time, i.e., $f(t)=D$.
S	initial inventory level at time $t=0$.
W	the capacity of owned warehouse (OW).
$\delta(t)$	the backlogging rate which is a decreasing function of the waiting time t, WLOG, we here assume that $\delta(t)=e^{-\sigma t}$ where $\sigma \ge 0$, and t is the waiting time.
$Z_o(t)$	$\alpha_o\beta_o(t-\gamma_o)^{\beta_o-1}$, $\alpha_o>0$, $\beta_o>0$, $t\ge\gamma_o\ge 0$, the deterioration rate at time t, where α_o, β_o, γ_o are scale, shape, location parameters of Weibull distribution in OW.
$Z_r(t)$	$\alpha_r\beta_r(t-\gamma_r)^{\beta_r-1}$, $\alpha_r>0$, $\beta_r>0$, $t\ge\gamma_r\ge 0$, the deterioration rate at time t, where α_r, β_r, γ_r are scale, shape, location parameters of Weibull distribution in RW.

Notice that (1) If β_o, $\beta_r>1$, then the deteriorating rate increases with time, e.g., fish and vegetables. (2) If β_o, $\beta_r<1$, then the deteriorating rate decreases with time, e.g., the light bulb in which the initial deterioration rate maybe higher due to irregular voltages or handling. (3) If β_o, $\beta_r=1$, then the deteriorating rate is constant, e.g., the electronic products. For detail, see Walpole and Myers (1978) and Berrettoni (1964).

r	the inflation rate.
c_o	the replenishment cost per order, including the transportation cost from OW to RW.