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# A three layer multi-item production–inventory model for multiple suppliers and retailers

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ABSTRACT

The paper deals with a production inventory model for various types of items where multiple suppliers, a manufacturer and the multiple non-competing retailers are the members of the supply chain. In this model, each supplier supplies only one type of raw material to the manufacturer. The manufacturer produces a finished item by the combination of certain percentage of the various types of raw materials. The manufacturer produces also multi-items and delivers them according to the demand of the different retailers. Finally, an integrated profit of the supply chain is optimized by optimal ordering lot sizes of the raw materials. A numerical example is provided to justify the proposed model.



## 1. Introduction

Supply chain management has taken a very important and critical role for any company in the increased globalization and competition in the market. The central aim of supply chain management translates the right products in the right quantities at the right place and right moment and minimum cost into the interrelated issues of customers' satisfaction. Nowadays, researchers have given importance to develop multi-echelon based supply chain inventory problems. In practice, people are becoming busy for their professional work and they want to do each and every non professional work at a minimum time. So, multi-retailing shops are increased slowly but surely. Consequently, marketing and manufacturing strategies are also changed with time. There are many products (electrical goods, auto mobile industry, foods product, drugs, etc.) which are produced in a factory but the production materials are collected from different places. Now, coordinating multiple raw material suppliers, manufacturing factory, multi-item, and multi-retailing shops in a supply chain is a big challenge to the researchers as well as to the practitioners.

Muckstadt (1973) studied a mathematical inventory model with a multi-item, multi-echelon, and multi-indenture system for recoverable items. Their discussion was limited to two echelon multi-item systems and they derived the logistics relationship between an assembly and its subassemblies and computed spare stock levels for both echelons for the assembly and subassemblies with explicit consideration. Graves (1979) developed lot scheduling problem in single machine for deterministic demand of multi-products in which the single production facility produced $n$ products individually. Eppen and Martin (1987) imposed a useful taxonomy on production scheduling problems and developed alternative formulations for a wide variety of problems within the taxonomy. Benton (1991) developed a quantity discount inventory model where multiple items, resource limitations and multiple suppliers are considered. Also, he considered an efficient heuristic programming procedure for evaluating alternative discount schedules. Silver et al. (2001) introduced an inventory model with multiple end items, each facing uncertain demand in a single period of interest but there is a stock of convertible units that can be transformed into any of the end items at costs which are dependent on the specific end items. Bhattacharya (2005) developed a multi-item inventory model for deteriorating items with a linear stock dependent demand rate. Brandimarte (2006) developed a stochastic version of the classical multi-item capacitated lot-sizing problem where a multi-stage mixed-integer stochastic programming model was discussed for uncertain demand. Hill and Pakkala (2007) studied a multi-item inventory system with random customers' orders where each order specifies a list of items. They also assumed that an item on an order is independent of what other items may or may not be included, subject to at least one item being listed. Haksever and Moussourakis (2008) presented an inventory system involving multiple products with known and constant independent demand, instantaneous replenishment, and constant lead times where no shortages are allowed. They also assumed that a single supplier exists for each product and offers incremental quantity discounts. Caggiano et al. (2009) developed a multi-item, multi-echelon distribution system with time based service level requirements where fill rates of the channel were computed accurately and efficiently. They presented a practical method for computing channel fill rates quickly that does not compromise with accuracy. Tsao (2010) discussed a multi-echelon multi-item channels with supplier's credit period and

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retailer's promotional effort. He analyzed two trade allowances, effort cost sharing and cash discount from the perspectives of the individual and channel. Taleizadeh et al. (2011) studied a multi-buyer multi-vendor supply chain problem with several products, capacity constraint of buyer and warehouse limitation of vendor. They assumed that the demand of each product is a stochastic variable and a uniform distribution follows and the lead-time of receiving products from a vendor to a buyer varies linearly with respect to the order quantity of the buyer. Li et al. (2011) derived a solution procedure of a multi-item capacitated dynamic lot-sizing problem considering both single-level and multi-level cases. They assumed that each item faces a series of dynamic demands, multiple items in each period and sharing limited production resources. Kamali et al. (2011) presented a multi-objective mixed integer nonlinear programming model to coordinate the system of a single buyer and multiple vendors under discount policy for the vendors. Sana (2011) developed an integrated multi-echelon production–inventory model of perfect and imperfect quality products where supplier, manufacturer and retailer are the members of the chain. There are some other interesting papers Sarkar (2011, 2012a, 2012b, 2012c) which are related to production inventory model. Tsao and Sheen (2012) studied a multi-item supply chain model with credit periods and weight freight cost discounts, from both the individual and channel perspectives. Pal et al. (2012a) introduced a multi-level inventory model considering perfect and imperfect quality items, product reliability and reworking of defective items in the environment of supply chain management. There are several interesting and relevant papers related to multi-echelon supply chain model such as Ben-Daya and Al-Nassar (2008); Cárdenas-Barrón (2007); Khouja (2003); Leung (2009, 2010); Sarker et al. (2008); Glock (2011), etc. Recently, Pal et al. (2012b) developed a multi-item revenue restricted inventory model considering selling-price and price break dependent demand rate.

In this model, the authors study a three-layer multi-item supply chain involving multiple suppliers, manufacturer and multiple retailers where each finished product is produced by the combination of the fixed percentage of various types of raw materials and each raw material supplier can supply only one material. Here, we consider that the manufacturer delivers finished products to the multiple retailers where each of the retailers sells their multiple products according to their demand in the market. Overall, the total integrated profit of the supply chain is evaluated and is optimized with respect to ordering lot sizes of the raw materials.

## 2. Fundamental assumptions and notation

The following assumptions and notations are used to develop the proposed model:

### 2.1. Assumptions

The following assumptions are made to develop the model:

(i) The model is developed for multiple ($m$) items.
(ii) There are multiple suppliers ($n$), one manufacturer and multiple retailers ($l$).
(iii) All the demands from the members of the supply chain are deterministic.
(iv) Manufacturer produces multiple items, one of each item can be made by the combination of the fixed percentage of various types of raw materials.
(v) Holding cost for the products, raw materials and members of the chain is different.
(vi) Each of the retailers collects the various types of finished product from the manufacturer according to their demand of the products.
(vii) Joint effect of the suppliers, manufacturer and the retailers is considered in the model.
(viii) Shortages at each stage are not allowed.

### 2.2. Notations

For suppliers $i = 1, 2, \ldots, n$, for products $j = 1, 2, \ldots, m$ and for retailers $k = 1, 2, \ldots, l$ are considered and the following notations are used throughout the paper to develop the model:

*Symbol denotation*

| | |
|---|---|
| $P_{s_i}$ | $i^{th}$ supplier faces the rate of demand from the manufacturer. |
| $Q_i$ | Ordering lot size for the $i^{th}$ supplier. |
| $T_{s_i}$ | Cycle length for the $i^{th}$ supplier. |
| $h_{s_i}$ | Holding cost per unit item per unit time for the $i^{th}$ supplier. |
| $w_{s_i}$ | Unit selling price for the product of $i^{th}$ supplier. |
| $C_{s_i}$ | Unit purchasing price for the product of $i^{th}$ supplier. |
| $\alpha_{ji}$ | Percentage of $i^{th}$ supplier's raw material to produce $j^{th}$ product. |
| $P_j$ | Production rate of the $j^{th}$ product. |
| $T_{p_j}$ | Production run-time of $j^{th}$ product. |
| $h_{m_j}$ | Holding cost per unit item per unit time for the $j^{th}$ product of the manufacturer. |
| $w_{m_j}$ | Unit selling price for the $j^{th}$ product of the manufacturer. |
| $\beta_{kj}$ | Percentage of demand of $j^{th}$ product to satisfy the demand of the $k^{th}$ retailer. |
| $T_{m_j}$ | Cycle length for the $j^{th}$ product for the manufacturer. |
| $C(P_j)$ | Unit production cost for the $j^{th}$ product for the manufacturer. |
| $L_j$ | Fixed cost like labor, energy and technology cost. |
| $\gamma_j$ | The variation constant of tool/die costs. |
| $D_{m_j}$ | $j^{th}$ product of manufacturer faces the rate of demand from the retailers. |
| $T_{r_{kj}}$ | Time for collecting $j^{th}$ product from manufacturer for the $k^{th}$ retailer. |
| $D_{c_{kj}}$ | $k^{th}$ retailer faces the rate of demand for the $j^{th}$ product from the customers. |
| $T_{kj}$ | Cycle length for the $j^{th}$ product for the $k^{th}$ retailer. |
| $h_{r_{kj}}$ | Holding cost per unit item per unit time for the $j^{th}$ product of the $k^{th}$ retailer. |
| $w_{r_{jk}}$ | Unit selling price for the $j^{th}$ product for the $k^{th}$ retailer. |

## 3. Mathematical formulation and analysis of the model

In this section, we develop a multi-item multi echelon production inventory model where multiple suppliers, one manufacturer and multiple retailers are the members of the chain. The manufacturer orders various types of raw material to the suppliers while each supplier can supply only one type of raw material. The manufacturer produces multi-items where each item is produced by the combination of a certain percentage of the raw materials. Each retailer sells various types of products to the customers in the market. According to the environment of cities, the demand of the market of the products is different. According to the market demand, the retailers order the products to the manufacturer. We formulate the model by assuming that the inventory level of any stages does not fall into shortages (Fig. 1).

### 3.1. The suppliers' inventory model

We formulate this section with $n$ suppliers where each supplier supplies only one raw material. Without loss of generality, we assume that $i^{th}$ supplier supplies $i^{th}$ raw material. The manufacturer orders $Q_i$ quantity of $i^{th}$ raw material at the beginning of the production process and collects at a rate $P_{s_i}$ from the $i^{th}$ supplier (Fig. 2). The governing differential equation, in this stage, is

$$\frac{dI_{s_i}(t)}{dt} = -P_{s_i}, \text{with } I_{s_i}(0) = Q_i \text{ and } I_{s_i}\left(T_{s_i}\right) = 0, 0 \le t \le T_{s_i}, \text{for } i = 1, 2, \ldots n. \quad (1.1)$$