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How to order and transship in multi-location inventory systems: The simulation optimization approach

Christian A. Hochmuth [a,*], Peter Köchel [b]

[a] Industrial Engineering, Bosch Rexroth AG, 97816 Lohr am Main, Germany
[b] Department of Computer Science, Chemnitz University of Technology, 09107 Chemnitz, Germany

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ABSTRACT

The efficient control of multi-location inventory systems with lateral transshipments has become increasingly important. However, existing models are analytically solvable only under simplifying assumptions. There is a variety of heuristics to find approximate solutions, but interdependencies between ordering and transshipment decisions for continuous time are not addressed. Thus, we suggest using simulation optimization. In our paper, we describe a widely adaptable simulation model for mapping highly complex multi-location inventory systems with lateral transshipments. By applying this model to a special case from the literature and to some examples, we show its validity and generality.

1. Introduction

With tougher competition, companies must improve service levels and reduce costs. Although these objectives seem contradictory, they may be reached. Spreading of service locations improves customer service and pooling of resources by lateral transshipments between locations decreases cost. The design and control of such multi-location systems is an important non-trivial task. Therefore, suitable mathematical models are needed – multi-location inventory models with lateral transshipments (MLIMTs) – to describe the following situation. A given number of locations have to meet a demand for some products during a defined planning horizon. Each location can get new product units either by ordering from an outside supplier or by transshipments from other locations. The problem arises to define such ordering and transshipment decisions that optimize given performance measures for the whole system.

At present a great variety of models and approaches exist dealing with this problem (see Paterson et al., 2011 for a review). The main difference of MLIMTs to classical inventory models is that lateral transshipments between locations are allowed. Depending on whether transshipments are organized after or before shortages occur, it is distinguished between emergency and preventive transshipments, respectively. In recent years some work is done to investigate the influence of transshipment policies on such performance measures as average cost, service level or mean supply delay (see e.g., Burton and Banerjee, 2005; Lee et al., 2007; Tiacci and Saetta, 2011, and the references therein). These papers assume a given order policy and use simulation to compare various heuristic transshipment policies, including the no transshipment situation. The most common and broadest investigated class of models which correspond to the joint optimization of ordering and transshipment decisions assume a single product, periodic review, independent and identically distributed demand through all periods, backlogging, complete pooling, emergency lateral transshipments at the end of a period, zero lead times, linear cost functions and the total expected cost criterion as performance measure. Approaches and results on these models can be found in Köchel (1998). MLIMTs generally do not allow analytical solutions due to transshipments. Transshipment decisions change the state of the system and thereby influence the ordering decisions. Thus, it is impossible to take the total consequences of an ordering decision into account. Approximate models and simulation are alternatives (e.g., Köchel, 1998, 2009; Robinson, 1990). Additional problems arise for continuous review models. One is to prevent undesirable forth-and-back transshipments. This is narrowly connected with the problem to forecast the demand during the transshipment time and the time interval elapsing from the release moment of a transshipment decision until the next order will arrive. Therefore, continuous review MLIMTs are usually investigated under several simplifying assumptions, e.g., two locations (Evers, 2001; Xu et al., 2003), Poisson demand (Kukreja et al., 2001), a fixed ordering policy without considering future transshipments (Minner et al., 2003), restriction to simple rules such as a

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* Corresponding author. Tel.: +49 163 3388579.
E-mail addresses: christian.hochmuth@boschrexroth.de (C.A. Hochmuth), peter.koechel@cs.tu-chemnitz.de (P. Köchel).

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one-for-one ordering policy (Kukreja et al., 2001) and an all-or-nothing transshipment policy (Evers, 2001) or the limitation that at most one transshipment with negligible time and a single shipping point during an order period is possible (Xu et al., 2003). However, the question for optimal order and transshipment policies remains unanswered in these papers. All models work with a given order policy and heuristic transshipment rules. In few cases simulation is used either for testing approximate analytical models (Minner et al., 2003; Xu et al., 2003) or for the definition of the best reorder point *s* for an (*s*,*S*) order policy (Kukreja and Schmidt, 2005) by linear search and simulation. Often the investigations are restricted to small-size models. Using sample-path-based optimization for order-up-to levels *S* and linear optimization to subsequently solve transshipments, allows calculating optimal order and transshipment policies for periodic review (Herer et al., 2006). However, it is assumed that interdependencies between orders and transshipments are negligible.

The MLIMT presented in this paper was developed subject to three main aspects. First, we wanted to connect periodic and continuous review approaches because in many practical situations periodic review is relevant for ordering decisions whereas continuous review applies to transshipments. Second, the model should surmount restrictions of existing models to ensure practical acceptance and should, therefore, be as general as possible. Third, a promising approach to find acceptable solutions is simulation optimization, coupling an MLIMT simulator with an optimization algorithm. The key advantage of the simulation optimization approach is that various performance measures can be optimized for in fact arbitrary MLIMTs. That allows us to investigate both the periodic and continuous review case, arbitrary demand processes as well as arbitrary ordering, demand satisfaction, pooling and transshipment modes. Also non-linear cost functions are feasible. Of course, the present version of the simulator has some limitations as, for instance, that only a single product is considered and transportation capacities are assumed to be infinite. However, the model presented in this paper is a valid proof of concept. Our contribution is to show the applicability of simulation optimization to highly complex MLIMTs and to describe a simulation model that generalizes common existing approaches. As a result, this model can be used to solve a wide range of special cases.

The paper is organized as follows. After a brief discussion of simulation optimization in Section 2, we describe multi-location inventory models in general and in terms of our implementation in Section 3. By investigating a special case from the literature, we validate our model in Section 4. Results for numerical examples are discussed in Section 5 and followed by a conclusion in Section 6.

2. Simulation optimization

The general idea of simulation optimization is outlined in Fig. 1. For a given decision problem, an optimizer proposes candidate solutions. Using results of simulation experiments, the performance of these candidate solutions is estimated. On the basis of the estimated performance, the optimizer decides to accept or reject the current decisions. Acceptance stops whereas rejection continues the search process. For an overview on simulation optimization approaches, we refer to Fu et al. (2005).

As seen from Fig. 1, simulation optimization is based upon two main elements—a simulator for the system to be investigated and an optimizer that finds acceptable solutions. Generic simulators and optimizers are compatible. This approach is, therefore, in principle suited for the solution of optimization problems from all fields of human activity.

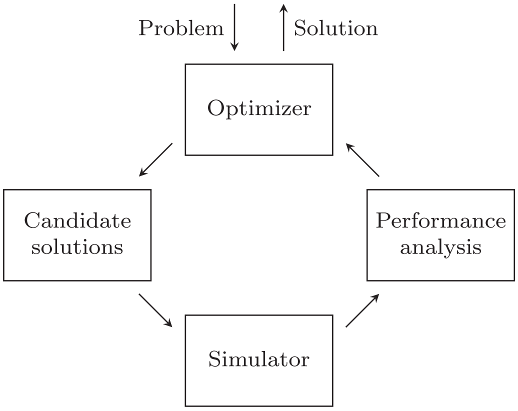

Fig. 1. Simulation optimization.

In the past, we implemented different applications especially to inventory problems (Kämpf and Köchel, 2006; Köchel, 2009; Arnold and Köchel, 1996; Köchel and Nieländer, 2005). In most cases, we used genetic algorithms due to important advantages. They can be designed and tested independently of the application domain. In fact, genetic algorithms are suitable for very general optimization problems, leave local optima and find the global optimum. Finally, they require little information and tend to deal excellently with the random output of simulation experiments. As various building blocks for genetic algorithms and other heuristics are available in the packages CHAOS (Kämpf, 2009) and CHEOPS (Nieländer, 2009), we focus on the design and implementation of a suited simulator.

3. Multi-location inventory models

A simulation model can represent any real system with arbitrary accuracy. However, our objective is far from over-sizing a simulator for all multi-location inventory systems but is to describe a simulator which is capable of finding efficient solutions for the optimal control of an important class of models in reasonable time. Additionally, that class should be broader than the models hitherto investigated. From a design viewpoint, it is reasonable to discuss the most important features of a general model first.

3.1. Features of a general model

The essential elements of a general multi-location inventory model with lateral transshipments are presented below.

Number N of locations. With respect to the analytical tractability, the cases $N=2$ and $N>2$ can be distinguished, while the latter is solvable only under simplifying assumptions.

Number of products. There may be a single product or a finite number of products. In the latter case, a substitution order between products represents one possibility to reduce complexity. If fixed costs for periodic and transshipment orders are zero, infinite capacities are assumed and demand is independent for all products, it is a viable to investigate the model sequentially.

Ordering mode. The ordering mode defines when to order, i.e., the *review scheme*, and what *order policy* to use. The review scheme defines the time moments for ordering. Periodic or continuous review is possible. Under the periodic review scheme, the planning horizon is divided into periods. Usually the ordering policy is defined by their type and corresponding parameters (e.g., order-up-to, one-for-one, (*s*,*S*), (*R*,*Q*)).