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An inventory model with variable demand, component cost and selling price for deteriorating items

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ABSTRACT

In this paper, we develop an economic order quantity (EOQ) model for finite production rate and deteriorating items with time dependent increasing demand. The component cost and the selling price are considered at a continuous rate of time. The objective of this model is to maximize the total profit over the finite planning horizon. We also want to find the integral number of orders in the finite planning horizon. A numerical example, graphical representations and sensitivity analysis are given to illustrate the model.

1. Introduction

In the classical inventory model, Harris (1915) considered an EOQ model with constant demand rate. Later, this model was discussed by Wilson (1934). Silver and Meal (1969) extended the EOQ model for varying demand rate. Donaldson (1977) derived an inventory model for linear trend in demand. Other researchers such as Buchanan (1980), Silver and Peterson (1985), Goyal et al. (1992), Teng (1996) and Lo et al. (2002) studied an inventory model with a linear trend in demand. In most of the above papers linear trend in demand or exponentially increasing demand rate were assumed. In reality, the demand rate for items may depend on time. Hsu and Li (2006) presented an optimal delivery service with time dependent consumer demand. Later, many researchers such as Banerjee and Sharma (2010) and Sarkar et al. (2011) developed the inventory model with different types of time dependent demand.

All of the above models were derived without deterioration. Deterioration is defined as decay, evaporation and loss of utility of a commodity that results in the decreasing usefulness from the original condition. Also, it is considered by the damages when the items are broken or loose their marginal value due to accumulated stress, bad handling etc. Ghare and Schrader (1963) formulated a model with exponentially decaying inventory. Shah and Jaiswal (1977) mentioned an order-level inventory model for a system with constant rate of deterioration. Aggarwal (1978) derived a note on an order-level inventory model for a system with constant rate of deterioration. Dave and Patel (1981) discussed inventory models with time proportional demand and deterioration. Dave (1986) addressed an order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replenishment. Bahari-Kashani (1989) discussed a replenishment schedule for deteriorating items with time proportional demand. Several researchers like Goswami and Chaudhuri (1991), Hariga (1995), Sarker et al. (1997), Jamal et al. (1997), Yan and Cheng (1998), Mandal and Pal (1998), Giri et al. (2000), Liao et al. (2000), Chang et al. (2001), Goyal and Giri (2003), Khanra and Chaudhuri (2003), Arcelus et al. (2003), Sana and Chaudhuri (2004) and Ouyang et al. (2005) developed different types of inventory models for different forms of deteriorating items with time dependent demand. Based on their models, Roy (2008), Sana (2008) and Lee and Hsu (2009) investigated order level inventory models with different forms of deteriorating items for time varying demand. Khanra et al. (2011) investigated an EOQ model for deteriorating items with time dependent quadratic demand. Widyadana et al. (2011) formulated an economic order quantity model for deteriorating items and planned back order level. Later, Sarkar (2012) obtained an EOQ model with time varying deterioration rate.

Thursby et al. (1986) addressed an inventory model with selling price. Lev and Weiss (1990) produced an inventory model with cost changes. Erel (1992) considered an EOQ model with price changes. Gascon (1995) surveyed an inventory model with cost changes for finite horizon. Wee (1995), Khouja and Park (2003), Goyal and Cárdenas-Barrón (2003), Teunter (2005) derived economic order quantity model under continuous price decrease. Several researchers like Cárdenas-Barrón (2006a, 2006b), Smith et al. (2007), Cárdenas-Barrón (2007), Mishra and Mishra (2008), Cárdenas-Barrón (2008), Smith et al. (2009), Cárdenas-Barrón

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(2009a, 2009b), Dye and Hsieh (2010), Sana (2011), Cárdenas-Barrón (2011), Yang et al. (2011), Cárdenas-Barrón et al. (2011), Lu et al. (2011), Naimzada and Ricchiuti (2011), Pal et al. (2012), and Sana (2012a, 2012b) obtained EOQ models with different types of practical assumptions.

In this paper, an effort has been made to develop an EOQ model with time dependent demand, component cost and product selling price for deteriorating items. This model is an extension of the concept of the model of Yang et al. (2011). The component cost and product selling price are increased day by day due to high design technology, reliability, inflations. Therefore, it is better to consider them as the increasing value rather than the decreasing value. Hence, the cost function and selling price are assumed inclined at a continuous rate per unit time which differs our paper from Yang et al. (2011) and many previous research models. Most of them considered reduced price and the corresponding demand is either constant or exponential dependent. But, in our model the demand is assumed as continuously increasing with respect to time quadratically. We consider deterioration of the product to make this model more realistic. The associated profit function is calculated and is optimized with respect to the integer number of orders in the entire planning horizon (Fig. 1). A numerical example, graphical illustration and sensitivity analysis are used to illustrate the model (Table 1).

2. Notation and assumptions

The following notation and assumptions are considered to develop the model:

Notation:

n	integral number of orders in the entire planning horizon which is our decision variable
Q_{i-1}	lot size during ith cycle, $i=1,2,....,n$
t_i	time point when the inventory level of ith cycle drops to zero
T	length of the replenishment interval
k	production rate (units/unit time)
$d(t)$	demand rate (units/unit time), where $d(t)=\xi_1+\xi_2 t+\xi_3 t^2$; $\xi_1, \xi_2, \xi_3>0$
C	component cost ($/units/unit time), where $C=C_0(1+r_c)^t$, C_0 is the component cost ($/units/unit time) when $t=0$, r_c is the incline-rate of component cost ($/units/unit time), $C_0>0$
S	selling price ($/units/unit time), where $S=S_0(1+r_s)^t$, S_0 is the selling price ($/units/unit time) when $t=0$, $S_0>0$
r_s	incline-rate of selling price ($/units/unit time)
H	length of the planning horizon
C_1	ordering cost ($/units/unit time)
C_2	holding cost ($/units/unit time)
$I(t)$	inventory level at time t
NP	net profit in the planning horizon

Assumptions:

1. The production rate k is finite.
2. Component cost and product selling price to the end consumer incline at a continuous rate per unit time.
3. Demand rate $d(t)$ is continuous and assumed to be quadratic function of time where $d(t)=\xi_1+\xi_2 t+\xi_3 t^2$; ξ_1, ξ_2, $\xi_3>0$.
4. Total planning horizon is finite.
5. Production rate or replenishment rate is greater than the demand of the produced product i.e. $k>d(t)$.
6. Shortage is not allowed as production rate is greater than the demand.
7. Lead time is assumed to be constant.
8. Deterioration rate θ is in constant, $0<\theta<1$.

3. Mathematical model formulation

Here, we consider the model for a fixed replenishment interval. We observe from the cycle period $[0,t_1]$ that production starts from $t=0$ and reaches at the point Q_0. From that point, the inventory level decreases with demand and drops at $t=t_1$. During the time $t=t_i$, inventory in ith cycle depletes to zero. The main aim of this problem is to calculate optimal values of integer n such that the total net profit is a maximum value.

For this model, we take

$$T=H/n \tag{1}$$

and

$$t_i=iT, i=1,2, ...,n. \tag{2}$$

The governing differential equation of the model is

$$\frac{dI(t)}{dt}=k-\theta*d(t), (i-1)T\le t\le iT, i=1,2, ...,n. \tag{3}$$

Using the boundary condition, $I(t)=0$ when $t=iT$. One has

$$I(t)=(k-\theta\xi_1)(t-iT)-\frac{\xi_2\theta}{2}\left(t^2+T^2\right)-\frac{\xi_3\theta}{3}\left(t^3+iT^3\right) \tag{4}$$

The lot size during the ith cycle is $I(t)$ when $t=(i-1)T$. We get,

$$Q_{i-1}=(\theta\xi_1-k)T-\frac{\xi_2\theta}{2}(1-2i)T^2-\frac{\xi_3\theta}{3}(2+3i)T^3 \tag{5}$$

For ith cycle, the unit component cost is $C_0(1+r_c)^{(i-1)T}$ and the holding cost is

$$\begin{aligned}HC_i&=\int_{(i-1)T}^{iT}C_2C_0(1+r_c)^{(i-1)T}I(t)dt\\&=C_2C_0(1+r_c)^{(i-1)T}\left[(\theta\xi_1-k)\frac{T^2}{2}-\frac{\xi_2\theta T^3(1-3i)}{6}-\frac{\xi_3\theta T^4}{12}(5+4i)\right]\end{aligned} \tag{6}$$

Total holding costs of the system HC are

$$HC=C_2C_0\left[(\theta\xi_1-k)\frac{T^2\xi_4}{2}-\frac{\xi_2\theta T^3}{2}\left(\frac{\xi_4}{3}+\frac{1}{(r_c)^{2T}}\right)-\frac{\xi_3\theta T^4}{3}\left(\frac{5\xi_4}{4}-\frac{1}{(r_c)^{2T}}\right)\right] \tag{7}$$

where $\xi_4=\frac{(1-r_c)^{nT}-1}{(1-r_c)^T-1}$.

Component cost during the ith cycle, PC_i is the product of Q_{i-1} and an unit component cost $C_0(1+r_c)^t$ at $t=(i-1)T$. Which implies,

$$PC_i=C_0(1+r_c)^{(i-1)T}\left[(\theta\xi_1-k)T-\frac{\xi_2\theta}{2}(1-2i)T^2-\frac{\xi_3\theta}{3}(2+3i)T^3\right] \tag{8}$$

Component cost in entire planning horizon, PC is the summation of n cycles of Eq. (8). One has

$$PC=C_0\left[(\theta\xi_1-k)T\xi_4-\frac{\xi_2\theta T^2}{2}\left(\xi_4-\frac{2}{(r_c)^{2T}}\right)-\frac{\xi_3\theta T^3}{3}\left(2\xi_4+\frac{3}{(r_c)^{2T}}\right)\right] \tag{9}$$