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Ergodic distribution for a fuzzy inventory model of type (s,S) with gamma distributed demands

Tahir Khaniyev [a,c,*], I. Burhan Turksen [a,b], Fikri Gokpinar [d], Basak Gever [a]

[a] Department of Industrial Engineering, TOBB University of Economics and Technology, 06560 Sogutozu, Ankara, Turkey
[b] Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8
[c] Institute of Cybernetics, Azerbaijan National Academy of Sciences, F. Agayev Street 9, AZ1141 Baku, Azerbaijan
[d] Department of Statistics, Gazi University, 06500 Teknikokullar, Ankara, Turkey

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ABSTRACT

In this study, a stochastic process $(X(t))$, which describes a fuzzy inventory model of type (s,S) is considered. Under some weak assumptions, the ergodic distribution of the process $X(t)$ is expressed by a fuzzy renewal function $U(x)$. Then, membership function of the fuzzy renewal function $U(x)$ is obtained when the amount of demand has a Gamma distribution with fuzzy parameters. Finally, membership function and alpha cuts of fuzzy ergodic distribution of this process is derived by using extension principle of L. Zadeh.

1. Introduction

Many problems of the inventory, stock control and reliability theories can be reduced to investigation of a semi-Markovian inventory model of type (s,S). In current literature, probability and numerical characteristics of these models are investigated under different distributions of random variables which are interpreted as the arrival times and demands. For example, in the studies of Aliyev, Khaniyev, and Kesemen (2010), Khaniev and Atalay (2010) and Khaniyev and Aksop (2011) the stationary characteristics of the models of type (s,S) are investigated for Gamma, Beta and Triangular distributions. These distributions contain various parameters. These parameters can be estimated by using the point and interval estimation methods. However, there can be fuzziness in datasets. Recall that, there are two kinds of uncertainty: randomness and fuzziness. Probability is traditionally used in modeling uncertainty under randomness. On the other hand, fuzziness introduced by Zadeh (1965) provide a different approach to treating uncertainty. Real models that contain uncertainty cannot be described adequately by using only randomness. From a practical viewpoint, the fuzziness and randomness in a process are often mixed with each other. Therefore, we need to combine these two kinds of uncertainty and use them together in practical applications. Thus, real models should be investigated with fuzzy logic to obtain more adequate results. For this reason, in this study, an inventory model of type (s,S) with a fuzzy parameter is considered and expressed by means of a semi-Markov process $(X(t))$, where $0 \leqslant s < S < \infty$.

There are many interesting studies in literature on the classical inventory models of type (s,S) and fuzzy renewal-reward processes (see, for example, Nasirova, Yapar, & Khaniyev, 1998; Gavirneni, 2001; Derieva, 2004; Artalejo, Krishnamoorthy, & Lopez-Herrero, 2006; Khaniev & Atalay, 2010; Khaniyev & Aksop, 2011 and so on). As well, in literature, there are some interesting studies devoted to the stochastic models with the application of the fuzzy logic approach (see, for example, Zadeh, 1968; Popova & Wu, 1999; Hwang, 2000; Dozzi, Merzbach, & Schmidt, 2001; Buckley, Feuring, & Hayashi, 2002; Zhao & Liu, 2003; Buckley & Eslami, 2004; Zhao, Tang, & Yun, 2006; Hong, 2006; Zhao & Tang, 2006; Li, Zhao, & Tang, 2007; Wang & Watada, 2009; Shen, Zhao, & Tang, 2009; Wang, Liu, & Watada, 2009; Li, 2011; Lia, 2011; Hwang & Yang, 2011; Wang, 2011, etc.).

For example, Popova and Wu (1999) have studied random renewal-reward process with random inter-arrival times and fuzzy rewards. Hwang (2000) investigated a renewal process in which the inter-arrival times are assumed as independent and identically distributed fuzzy random variables, and proved an almost sure convergence theorem with the probability measure for the renewal rate. Dozzi et al. (2001) provided a limit theorem for counting renewal processes indexed by fuzzy sets. Zhao and Liu (2003) discussed a fuzzy renewal process defined by a sequence of positive fuzzy variables and established the fuzzy elementary renewal theorem and renewal-reward theorem. Hong (2006) discussed a renewal process in which inter-arrival times and rewards are depicted by L–R fuzzy numbers under t-norm-based fuzzy operations. Zhao and Tang (2006) derived some other properties of fuzzy

* Corresponding author at: Department of Industrial Engineering, TOBB University of Economics and Technology, 06560 Sogutozu, Ankara, Turkey.
E-mail address: tahirkhaniyev@etu.edu.tr (T. Khaniyev).

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random renewal processes and obtained Blackwell's renewal theorem and Smith's key renewal theorem for fuzzy random inter-arrival times. Li et al. (2007) introduced the fuzzy random variable into delayed renewal processes and discussed a fuzzy random delayed renewal process as well as a fuzzy random equilibrium renewal process which is a special case of the former. Wang et al. (2009) discussed fuzzy random renewal process and renewal-reward process under hybrid uncertainty of fuzziness and randomness, and applied them to queuing system. Wang and Watada (2009) have discussed a renewal-reward process with fuzzy random inter-arrival times and rewards under the independence with t-norms, and they have derived a new fuzzy random renewal-reward theorem for the long-run expected average reward. Shen et al. (2009) and Lia (2011) established an alternating renewal process with two states: on times and off times. Hwang and Yang (2011) have created the extended elementary renewal theorem, renewal-reward theorem, Wald's equation for fuzzy renewal processes and fuzzy renewal-reward processes with fuzzy random inter-arrival times, fuzzy random rewards and fuzzy random stopping times. They have also considered some of their applications. Wang (2011) has investigated the mixture inventory control system in which the lead time and demands in different cycles are independent and identically distributed random variables. Moreover, the backorder rate, ordering cost, shortage penalty cost and marginal profit per unit in different cycles are independent identically distributed fuzzy variables, respectively. Also a mathematical formulation about the expected annual total cost is presented, based on the fuzzy random renewal-reward theory.

As it is seen above, there are numerous studies about renewal-reward process with fuzzy parameters in the current literature. However, inventory models of type (s,S) which are important applications of those theories are not investigated sufficiently. Therefore, in this study, a semi–Markovian inventory model of type (s,S) $(X(t))$ is constructed and the process $X(t)$ is investigated under the assumption that demands are random variables having a Gamma distribution with fuzzy parameters.

The model. It is assumed that stock level $(X(t))$ in a depot at the initial time $(t = 0)$ is $X(0) = X_0 = S$; $0 \leqslant s < S < \infty$. Furthermore, it is assumed that at random times $T_1, T_2, \ldots, T_n, \ldots,$the stock level $(X(t))$ in the depot decreases according to $\eta_1, \eta_2, \ldots, \eta_n, \ldots,$ until the stock level falls below the predetermined control level s. Therefore, the stock level in the depot changes as follows:

$$X(T_1) \equiv X_1 = S - \eta_1;\ \ X(T_2) \equiv X_2 = S - (\eta_1 + \eta_2); \ldots;\ \ X(T_n) \equiv X_n = S - \sum_{i=1}^{n} \eta_i; \ldots$$

Here, η_n represents the quantity of the n^{th} demand. The demands occur at the random times $T_n = \sum_{i=1}^{n} \xi_i$, $n = 1, 2, \ldots$ and the stock level of depot decreases at these times according to quantities of demands $\{\eta_n\}$, $n \geqslant 1$. This variation of the system continues until the certain random time τ_1, where τ_1 is the first crossing time of the control level s $(s \geqslant 0)$. At time τ_1, the process immediately comes to level S, again. Thus, the first period has been completed. Afterwards, the system continuous its variation from initial state S similar to the first period. When the stock level is decreased to below the control level s for the second time, the stock level is immediately brought to level S and the process changes as similar to the previous periods.

Let's illustrate this model as a real-world example.

The real-world model. A company operating in the energy sector produces, stores, fills and distributes liquefied petroleum gas (LPG). Domestic LPG distribution is carried out through pipelines and land transport. Where there is no pipeline installation, gas is distributed through land transport. LPG is carried from the LPG production center (a city in Turkey) to the 30 dealers by tankers with the capacities of 22 m^3 (approximately 10–11 tons) and 35 m^3 (approximately 17–18 tons). The tankers are kept under surveillance with Global Positioning System (GPS) 24 h a day and seven days a week. After delivering the needed amount of gas to the dealer, if more than 10% of the capacity of the tanker is left over, the tanker waits in its position until the next order of any dealer. Each dealer has a storage capacity of $S = 30$ m^3. Random amounts of LPG (η_n) are sold from these storage tanks at random times (ξ_n). When at random moments τ_n, $n \geqslant 1$, the level of LPG in the tank of the dealer falls below the control level $s = S/5$, a demand signal is automatically sent online to the production center. As a response to this demand, the nearest tanker to the dealer is directed to the demanding dealer. If there is no tanker near to the dealer, a full tanker is sent from the production center and depot is filled to the level S. Note that to modeling the amounts of demands η_n cannot be identified just by randomness, but also with fuzziness.

Therefore, the aim of this study is to construct such a stochastic process mathematically, which expresses the model stated above, in order to investigate the ergodic distribution of the process with the use of fuzzy logic approach.

2. Construction of the process $X(t)$

We introduce two independent sequences of positive valued random variables $\{\xi_n\}$ and $\{\eta_n\}$.

$n \geqslant 1$, defined on some probability space $(\Omega, \mathcal{F}, P)$. Moreover, it is assumed that the variables in each sequence are independent and identically distributed. The random variables η_n, $n \geqslant 1$ are interpreted as the amount of demands; ξ_n, $n \geqslant 1$, are interpreted as the inter-arrival times between demands. Using the initial sequences $\{\xi_n\}$ and $\{\eta_n\}$, the renewal sequences $\{T_n\}$ and $\{Y_n\}$ are defined as follows:

$$T_n = \sum_{i=1}^{n} \xi_i,\ \ Y_n = \sum_{i=1}^{n} \eta_i,\ \ T_0 = Y_0 = 0, \quad n \geqslant 1.$$

In addition, the sequence of integer-valued random variables $\{N_n\}$ is constructed as follows:

$$N_0 = 0,\ \ N_1 = \min\{k \geqslant 1 : S - Y_k < s\},$$
$$N_{n+1} = \min\{k \geqslant N_n + 1 : S - (Y_k - Y_{N_n}) < s\}, \quad n \geqslant 1.$$

Moreover, we put

$$\nu(t) = \max\{k \geqslant 0 : T_k \leqslant t\}; \quad \tau_n = T_{N_n} = \sum_{i=1}^{N_n} \xi_i,\ \ n \geqslant 1;\ \ \tau_0 = 0.$$

Now, we can construct a desired stochastic process $X(t)$ as follows:

$$X(t) = S - (Y_{\nu(t)} - Y_{N_n}), \quad \text{if } \tau_n \leqslant t < \tau_{n+1},\ \ n \geqslant 0.$$

The random variables τ_n, $n \geqslant 0$ are interpreted as the passage times to the control level $s > 0$ by the process $X(t)$.

A sample trajectory of the process $X(t)$ is given below.

Main aim of this study is to calculate the membership function of the ergodic distribution of the process $X(t)$, when some parameters of the demand are fuzzy numbers.

3. Main results

First, let us give the classical result about this model before investigating the model as a part of fuzzy logic. Assume that $Q(x)$ denotes the ergodic distribution of the process $X(t)$, and $U_\eta(x)$ is the renewal function generated by the sequence $\{\eta_n\}$,$n \geqslant 1$. In Nasirova et al. (1998) [7], the ergodic distribution of the process $X(t)$ is found. Here, we state this result as the following proposition without proof.