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Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates

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ABSTRACT

This paper considers an inventory system with non-instantaneous deteriorating item in which demand rate is a function of advertisement of an item and selling price. This paper aids the retailer in maximizing the total profit by determining optimal inventory and marketing parameters. In contrast to previous inventory models, an arbitrary holding cost rate and arbitrary deterioration rate have been incorporated to provide general framework to the model. First, a mathematical model is formulated and then some useful theoretical results have been framed to characterize the optimal solutions. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions are also derived. An algorithm is designed to find the optimum solutions of the proposed model. Numerical examples are included to illustrate the algorithmic procedure and the effects of key parameters are studied to analyze the behavior of the model.

1. Introduction

In recent years, inventory problems for deteriorating items have been widely studied. Deterioration is defined as decay, change or spoilage such that the items are not in a condition of being used for its original purpose. Electronic goods, radioactive substances, grains, blood, alcohol, gasoline, turpentine are examples of deterioration items. For any business organization, it is major concern to control and maintain the inventories of deteriorating items.

A model with exponentially decaying inventory was initially proposed by Ghare and Schrader [1]. Covert and Philip [2] formulated model with variable deteriorating rate of two-parameter Weibull distribution. Philip [3] generalized this model by taking three-parameter Weibull distribution. After that many researchers such as Goyal [4], Raafat et al. [5], Wee [6], and others developed models on deteriorating items. A detailed review of deteriorating inventory literatures is given by Goyal and Giri [7]. There is a vast inventory literature on deteriorating items under different conditions, the outline which can be found in articles [8–12] and their references.

Price is one of the key factors which influence the demand of any type of product. In essence, the lower selling price raises the selling rate whereas the higher selling price has reverse effect. Abad [13] has made a distinction based on two assumptions: (1) annual demand for the item is fixed and the retailer only has to plan his procuring (i.e., lot-sizing) policy; (2) annual demand is a function of the price. In latter case, retailers are confronted with decisions regarding pricing and lot-sizing. Subsequently, several scholars have examined these two issues together in a number of studies under different conditions. Many related articles can be found in Kim and Lee [14], Khouja [15], Smith et al. [16–17], Kim and Bell [18], Gonzalez-Ramirez et al. [19], Yang et al. [20], Arcelus et al. [21], Jörnsten et al. [22] and their references. Accordingly, pricing and inventory policies are two major concerns for any business organization that deals with perishable items. In this context, Wee [23] established the joint pricing and replenishment policy for a deteriorating item with price elastic demand rate that decline over time. Abad [24] studied the optimal pricing and lot-sizing policies under conditions of perishibility and partial backordering. Wee [25] developed a replenishment policy for deteriorating items with price dependent demand and Weibull distribution deterioration. Mukhopadyay et al. [26–27] discussed pricing and lot-sizing problem for a product with a time proportional and two-parameter Weibull distribution deterioration rate respectively. Recently, Begum et al. [28]

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presented pricing and replenishment strategy for an item with three-parameter Weibull distribution deterioration rate. There are several interesting papers on joint pricing and replenishment policy for a deteriorating item such as [29–34] and so forth.

A common characteristic to the aforementioned articles is that the deterioration of items in inventory starts from the instant of their arrival. However, many items maintain freshness or original condition for a certain period of time. In other words, deterioration does not occur for particular period of time. Wu et al. [35] advocated this phenomenon and analyzed inventory model for non-instantaneous deteriorating items. Motivated by own work, Wu et al. [36] formulated and solved an inventory system with non-instantaneous deteriorating items and price-sensitive demand. Recently, Maihami and Kamalabadi [37] established joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. Ouyang et al. [38], Geetha and Uthayakumar [39] and Musa and Sani [40] have discussed inventory ordering policies for non-instantaneous deteriorating items under condition of permissible delay in payment. In the above cited papers on non-instantaneous deteriorating items, researchers have considered constant deterioration rate. However, deterioration depends on time and this phenomenon has been overlooked by the researchers. Note that the articles on non-instantaneous deteriorating items appeared up to now do not address the deterioration with duration of storage.

Apart from price, the other marketing parameter which affects the demand is advertisement. It is commonly seen that a product is promoted through the advertisement in the well-known print or electronic media or by other means to attract the customers. The purpose of this type of advertisement is to raise the demand of the product. According to Deane and Agarwal [41], the online advertising industry realized annual revenues estimated at over $26 billion, in the United States alone, in 2010. Banner advertising accounts for an estimated 23% of all online advertising revenues. Considering the effect of advertising and price variation on demand rate various inventory models have been proposed by scholars like Subramanyam and Kumaraswamy [42], Urban [43], Goyal and Gunasekaran [44], Pal et al. [45], Bhunia et al. [46] and so forth.

Again, in traditional inventory models, holding cost is known and constant. But holding cost may not always be constant. This is particularly true in the storage of deteriorating and perishable items such as food products. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like Naddor [47], Muhlemann and Valtis Spanopoulos [48], Weiss [49], Goh [50], Giri et al. [51], Alfares [52], Roy [53], Pando et al. [54], and so on.

Based on above discussion, the proposed study considers the inventory model for non-instantaneous deteriorating item to allow for: (1) demand rate is linked to both frequency of advertisement and the selling price, (2) general-type of deterioration and holding cost rates, and (3) a profit maximization objective. Such considerations make the study advantageous and provide a general framework that includes numerous previous studies such as in [25–28,36] as special cases. Some useful theoretical results have been derived to characterize the optimal solutions. An easy-to-use algorithm to find the optimum solution is developed. Numerical examples are provided to demonstrate the developed model and solution procedure. A rigorous analysis on optimal solution with respect to key parameters has been carried out and the results are discussed in detail.

The rest of the paper is organized as follows: In Section 2, the assumptions and notations which are used throughout the article are presented. In Section 3, mathematical model to maximize the total profit is formulated. Solution methodology comprising some useful theoretical results and algorithm to find the optimal solution is carried out in Section 4. Numerical examples are provided in Section 5 to illustrate the theory and the solution procedure followed by managerial implications. Finally, we draw a conclusion in Section 6.

2. Assumptions and notations

The following assumptions and notations have been used in developing the mathematical model in this article.

2.1. Assumptions

(1) The inventory system involves single non-instantaneous deteriorating item.
(2) Demand rate $D(A,p)$ is a function of marketing parameters with the frequency of advertisement (A) and the selling price (p). In this paper, we assumed power form of the selling price and the frequency of advertisement for demand function; i.e., $D(A,p)=A^{\eta}ap^{-b}$ where $a(>0)$ is the scaling factor, $b(>1)$ is the index of price elasticity, and η is the shape parameter, where $0 \le \eta < 1$.
(3) During the fixed period, γ, the product has no deterioration. After that the on-hand inventory deteriorate with variable rate $\theta(t)$, where $0<\theta(t)<1$.
(4) There is no replacement or repair of deteriorated units during the period under consideration.
(5) Shortages are not allowed.
(6) Replenishment rate is infinite and lead time is zero.
(7) The system operates for an infinite planning horizon.

2.2. Notations

K	The ordering cost per order.
γ	The length of time in which the product has no deterioration.
A	Frequency of advertisement per cycle.
G	Cost for each advertisement.
c	The purchasing cost per unit.
p	The selling price per unit.
$h(t)$	Unit holding cost per unit time at time t.
Q	The order quantity.
T	Length of replenishment cycle.
$\theta(t)$	The deterioration rate of the on-hand inventory over $[\gamma,T]$.
$I_1(t)$	The inventory level at time t $(0 \le t \le \gamma)$ in which the product has no deterioration.
$I_2(t)$	The inventory level at time t $(\gamma \le t \le T)$ in which the product has deterioration.
$Z(A,p,T)$	The total profit per unit time of inventory system.

3. Mathematical model

The inventory system evolves as follows: Q units of items arrive at the inventory system at the beginning of each cycle. Based on values of T and γ, two cases viz. $T \ge \gamma$ and $T \le \gamma$ arise (Fig. 1). We discuss each case in detail as follows.

Case 1. $T \ge \gamma$. In this case, the inventory level is declining only due to demand rate over time interval $[0,\gamma]$. The inventory level is reducing to zero owing to demand and deterioration during the time interval $[\gamma,T]$. The aforesaid process is repeated.