Int. J. Production Economics 143 (2013) 53–61

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An integrated production inventory model with raw material replenishment considerations in a three layer supply chain

Mohammed Ben-Daya *, Rami As'ad, Mohammed Seliaman

Department of Systems Engineering, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

ARTICLE INFO

Article history:
Received 1 January 2010
Accepted 22 October 2010
Available online 28 October 2010

Keywords:
Production planning
Inventory control
Supply chain coordination
Joint economic lot sizing

ABSTRACT

Most of the existing research has focused on a two stage single-vendor single-buyer supply chain. However, in reality, supply chain networks are more complex and involve more than just a vendor and a buyer. This paper deals with the joint economic lot sizing problem (JELP) in the context of a three stage supply chain consisting of a single supplier, single manufacturer and multi-retailers. The objective is to specify the timings and quantities of inbound and outbound logistics for all parties involved such that the chain-wide total ordering, setup, raw material and finished product inventory holding costs are minimized. In developing the model, the cycle time at each stage is set to be an integer multiple of that for the adjacent downstream stage. To bear a better resemblance to practice, shipments from a particular lot are allowed to take place during production and not after producing the whole lot. We employ derivative-free methods to derive a near closed form solution for the developed model. A numerical example is presented for illustrative purposes and a comparison to models established in the literature is also provided.

1. Introduction

For most companies, providing the customers with a better service at a reduced cost is one of the ultimate strategic goals. The production of highly diversified products with short life cycles such as computer parts, fashion clothes and some food items among many others as well as the remarkably high levels of competition pushes the different companies towards the integration of different production and inventory related decisions. Consequently, companies are realizing the necessity of having elevated levels of mutual understanding and better collaboration with their customers and suppliers alike. To remain competitive, firms can no longer operate as individual and autonomous entities but rather as an integral part of the supply chain.

The area of supply chain management (SCM) has gained a lot of interest from researchers as well as practitioners in the industry. In particular, the integrated single-vendor single-buyer problem, also called the joint economic lot sizing problem (JELP), has received a lot of attention in recent years as it represents the building block for the wider supply chain. Essentially, the retailer (buyer) observes a deterministic demand and orders lots from the manufacturer (vendor). The vendor satisfies this downstream demand through manufacturing the requested product in lots, where each produced lot is shipped to the buyer in batches. The problem is to find the number of shipments and size of each batch such that the joint manufacturer and retailer cost is minimized.

For a vertically integrated supply chain owned partially or jointly by the same company, such coordinated production–shipment policy provides valuable insights and optimal decisions that lead to global optimization. On the other hand, when individual entities are owned separately, such policy may not benefit all parties equally as some may encounter an increase in their costs and hence become less eager to depart from their locally optimized policies. In such situations, sharing those benefits resulting from the coordinated approach becomes a major issue. By using effective incentive systems such as accounting methods, transfer pricing schemes, quantity discount, etc., the objective of each partner can be aligned to that of the supply chain as a whole (Ganeshan, 1999; Li & O'Brien, 1999; Agrawal et al., 2004).

Most of the work related to JELP has been conducted in the context of a two layer supply chain consisting of a single vendor and a single buyer. Goyal (1977) suggested a lot-for-lot policy with the assumption of infinite production rate for the manufacturer. Later, Banerjee (1986) maintained the lot-for-lot policy for the more realistic case of a finite production rate. The lot-for-lot assumption was relaxed by Goyal (1988) where he assumed that the vendor ships the lot in a number of equal size shipments. Goyal (1995) developed a policy where the shipment sizes increase by a factor increasing geometrically. Hill (1997) generalized the latter model through considering the geometric growth factor as a decision variable. The optimal solution to the problem in its general form (i.e., without any assumptions regarding the shipment policy) was

* Corresponding author.
E-mail address: bendaya@kfupm.edu.sa (M. Ben-Daya).

0925-5273/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2010.10.024

obtained by Hill (1999). Goyal and Nebebe (2000) considered a policy where the first shipment is small and the following shipments are larger and of equal size. For a comprehensive review of the JELP, the reader is referred to Ben-Daya et al. (2008).

More recently, this problem has been extended to the case of a three layer supply chain. Khouja (2003) was the first to consider a three stage supply chain with one or more firms at each stage. He discussed three inventory coordination mechanisms among the members of the supply Chain: (1) The equal cycle time mechanism where all parties involved share a common cycle time. This corresponds to the lot-for-lot policy mentioned earlier. (2) The integer multipliers of the cycle time mechanism where the cycle time at each stage is set to be an integer multiplier of the cycle time of the adjacent downstream stage. (3) The integer power of two multipliers mechanism in which the cycle time at each stage is an integer power of two multiples of a basic cycle time. Khouja found out that the savings in going from the first mechanism to the second one is more significant as compared to the savings obtained once going from the second to the third coordination mechanism. Recently, Ben-Daya and Al-Nassar (2008) applied the idea of Lu (1995), which calls for making shipments from a production lot as it is being produced, to the three stage multi-customers supply chain using the integer multiplier of the cycle time mechanism. The authors showed that their policy results in a lower total cost as compared to the corresponding policy suggested by Khouja. Lee (2005) added a new dimension to the single vendor single buyer problem by setting the number of raw material shipments received by the vendor per cycle to be a decision variable. Thus, the raw material ordering cost was considered explicitly in the model. According to Lee (2005), models incorporating the raw material procurement and manufacturing setup are called integrated procurement-production (IPP) systems.

The purpose of this paper is to incorporate Lee's idea suggested in a two layer supply chain in the model developed by Ben-Daya and Al-Nassar (2008) for three layer supply chains. The supply chain we are dealing with consists of a single supplier, single manufacturer and multi-retailers. The raw material ordering cost for both the supplier and the manufacturer are included in the model, and the number of raw material shipments received by both parties in every cycle is a decision variable. A cost minimization model is derived along with an efficient solution algorithm that is based on the algebraic approach.

The remainder of this paper is organized as follows. Section 2 states the problem definition, notations and assumptions. The derivation of the mathematical model is detailed in Section 3 while the analysis of the model and the solution procedure are both provided in Section 4. The numerical example along with sensitivity analysis for key problem parameters is given in Section 5. Finally, Section 6 concludes the paper and highlights future research directions.

2. Problem definition, notations and assumptions

Consider a three-layer make-to-stock (MTS) supply chain involving a supplier, a manufacturer and multi-retailers. An example of such supply chain configuration is depicted in Fig. 1. The supplier receives the raw material from his supplier and transforms it to semi-finished products at a certain production rate (P_s). The manufacturer, in turn, receives those semi-finished items from the supplier in equal sized batches and transforms them to finished products at a rate of (P_m). The number of per cycle inbound and outbound shipments for both the supplier and the manufacturer is a decision variable. The finished products are shipped to the retailers in equal shipments and they are used by the retailers to fulfill end customers' demand. Shipments from a particular lot are allowed to take place during the production of

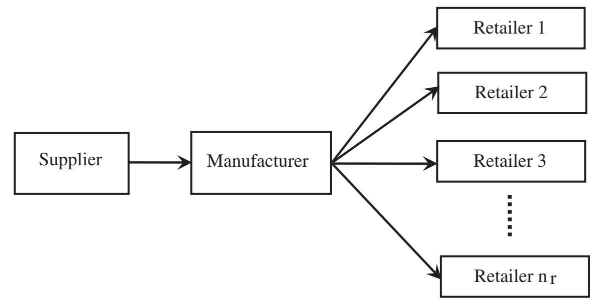

Fig. 1. A three layers multi-retailer supply chain.

that lot. Although the retailers receive shipments at the same timings in what is known as common replenishment epochs (Viswanathan and Piplani, 2001), the size of these shipments may vary from one retailer to another depending on each retailer's demand. The problem is to develop an integrated model specifying the ordering, production and shipment lot sizing policy, so that the chain wide total cost is minimized.

The mathematical model will be developed using the following notations:

P_s	production rate of the supplier
P_m	production rate of the manufacturer
D_s	demand rate of the supplier
D_m	demand rate of the manufacturer
$D_{r,j}$	demand rate observed by retailer j, $D_s=D_m=\Sigma_j D_{r,j}=D$
T_s	supplier's cycle time
T_m	manufacturer's cycle time
T	common basic cycle time for all retailers
K_1	integer multiplier of the manufacturer's cycle time, $T_s=K_1T_m(T_s=K_1K_2T)$
K_2	Integer multiplier of the retailers' cycle time, $T_m=K_2T(T_s=K_1K_2T)$
h_0	the per unit holding cost for the supplier's raw materials per unit time
h_s	the per unit holding cost for the supplier's finished product and manufacturer's raw materials per unit time
h_m	the per unit holding cost for the manufacturer's finished products per unit time
h_r	the per unit holding cost for the retailers per unit time
A_s	supplier's production setup cost
A_m	manufacturer's production setup cost
O_s	supplier's raw material ordering cost
O_m	manufacturer's raw material ordering cost
O_r	retailer's finished product ordering cost
m_1	number of raw material shipments received by the supplier within a cycle
m_2	number of raw material shipments received by the manufacturer within a cycle
n_r	number of retailers.
Q_s	the lot size received and produced by the supplier per cycle
G_s	the shipment size received by the supplier, $G_s=Q_s/m_1$
Q_m	the lot size received and produced by the manufacturer per cycle
G_m	shipment size received by the manufacturer, $G_m=Q_m/m_2$
G_r	shipment size received by all retailers
$G_{r,j}$	shipment size received by retailer j, $G_r=\Sigma_j^{n_r}G_{r,j}$
TC_s	total cost per unit time for the supplier
TC_m	total cost per unit time for the manufacturer
TC_r	total cost per unit time for all retailers
$TC\,(K_1, K_2, T, m_1, m_2)$	total cost per unit time for the whole supply chain