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Dilemma of rented warehouse and shelf for inventory systems with displayed stock level dependent demand

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ABSTRACT

Impact of display stock has been extensively discussed in literature in terms of stock dependent demand rate. In some recent papers a three-component inventory level dependent demand (as displayed stock level (DSL)) has been proposed. It has been assumed that demand is independent of instantaneous inventory level if inventory is above a certain level in a shelf. As the demand is independent of instantaneous inventory level, a retailer is always in dilemma whether inventory additional to that level will be kept in the shelf or to rent a new storage. In this paper we focus on this by developing three mathematical models. We show that the problem in choosing rented storage or shelf to keep additional inventory can always be eliminated (a) without the knowledge of optimal stocking points or (b) with the knowledge of any one optimal stocking point. Also, if there is enough opportunity to have an extra storage and a mild limit on inventory holding cost in the shelf is satisfied, then there is no need to display inventory in the shelf above the certain level. The results are illustrated by numerical examples and sensitivity analysis is carried out.

1. Introduction

Mass display of items in departmental stores and busy marketplaces has psychological impact on customers and it stimulates sales of some retail items. Motivated by this, researchers and practitioners have proposed several demand states to explore demand variability with respect to display stock level (DSL). Mainly, two types of demand patterns, e.g. initial display level dependent and instantaneous display level dependent, are observed in the literature. Instantaneous DSL dependent demand is further classified into two types, e.g. single-state and two-state. With two-state, demand is independent of DSL below a certain inventory level but there is no upper limit of DSL dependency of demand. This limitation is treated in the literature by imposing a restriction on capacity of shelf space. Some researchers in recent years have treated this drawback alternatively by proposing a three-state DSL dependent demand. The three-state DSL dependent demand is structured by adding a state, as upper limit of DSL dependency, to two-state DSL dependent demand. Inventory displayed above that limit in the shelf has no impact on customers. Although it's possible to achieve utmost benefit of display level dependency of demand as long as display level is above that limit, still a retailer is always in indecision whether inventory should be piled up in the shelf above that level or not.

In this paper, we discuss three models. Two models are based on the three-component DSL dependent demand (TCD from now on), where inventory is replenished above the limit of display dependency. In the first model inventory additional to that limit is stored in rented warehouse, whereas in the second model it is displayed in the shelf. The third model is based on the two-component DSL dependent demand (TWCD from now on). Concavities of all the models are verified. The key contribution of the paper is in responding to the following queries. First, under what circumstance the retailer would pile up inventory above the limit of DSL dependency of demand? That is, whether a retailer would choose two-state or three-state DSL dependent demand. Second, if inventory is replenished above the level of DSL dependency then would the inventory extra to it be stored in a rented warehouse or would it be displayed in the shelf?

The rest of the paper is organized as follows. In Section 2 inventory literature is reviewed for the DSL dependent demand and two warehouse inventory problems. Mathematical models are developed and are analyzed in Section 3. Section 4 deals with some numerical analysis of the proposed model. Finally Section 5 deals with summary and concluding remarks.

2. Literature review

Classical economic order quantity (EOQ) model was the first fundamental model of inventory literature where trade-off between variable

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holding cost and fixed setup cost was determined under static demand rate. But demand of physical goods, especially in a retail industry, is largely influenced by the product in the shelf. According to Levin et al. (1972) "at times, the presence of inventory has a motivational effect on people around it. It is a common belief that large piles of goods displayed in a departmental store leads the customers to buy more." Chang et al. (2006) have mentioned that an increase in shelf space for an item always induces more customers to buy it. This occurs because of its visibility, popularity or variety. To explore this, in the last three decades variability of inventory level dependent demand rate on the analysis of inventory system is described by researchers like Silver and Meal (1979), Phelps (1980), Silver and Peterson (1985), Ritchie and Tsado (1985), Mandal and Phaujdar (1989), Urban (1992), Goswami and Chaudhuri (1992), Pal et al. (1993), Padmanabhan and Vrat (1995), Baker and Urban (1998), Chang et al. (2006), Abbott and Palekar (2008), Goyal and Chang (2009), Sarkar et al. (2010), Chang et al. (2010), Sana (2010, 2011a,b, 2012) and others. Urban (2005) has mentioned "Operations management literature has recognized this motivating effect of inventory on demand, and models have been developed that incorporate this relationship." There is a vast literature on inventory level dependent demand and its overview can be found in the review article by Urban (2005). He has classified the models on inventory level dependent demand into two major categories—(i) initial inventory level dependent and (ii) instantaneous inventory level dependent and unified these two types of demand by considering a periodic review model. The explanation in the model of Urban (2005) for instantaneous inventory level dependent demand has been restricted to two components as assumed by Datta and Pal (1990), Dye and Ouyang (2005), Panda et al. (2008, 2009), Urban (1992) and others. But the main limitation of the two-component instantaneous inventory level dependent demand or simply instantaneous inventory level dependent demand is that, more stock in the shelf leads to more profit/lesser cost. Ultimately maximum system profit/minimum system running cost is achievable for an infinite amount of stock in shelf. As a result, in model incorporating instantaneous inventory level dependent demand pattern, the optimal order quantity is never found. The profit function/cost function increases/decreases with respect to the increment of stock in the shelf. Chang et al. (2006) have noted this problem for Dye and Ouyang's (2005) model and introduced an upper limit for displayed inventory level in the shelf. They have used the logic that, most retail outlets have limited shelf space and "too much piled up in everyone's way leaves a negative impression on buyer and employee alike" as stated in Levin et al. (1972). Panda et al. (2009) have also observed this for the two-component stock dependent demand with positive terminal inventory level and introduced a constraint as shelf space limitation to determine optimal ordering quantity. However, in some recent papers this limitation is treated by proposing a market oriented three-component demand rate that depends on displayed stock level (DSL). Pal et al. (2005, 2006) have proposed inventory models with three-component displayed stock level (DSL) dependent demand rate (TCD) incorporating frequency of advertisement and selling price of the product. They have claimed that "in competitive market place glamorous display of large number of products with the help of modern light and electric arrangement influences the customers to buy more. But in practice the demand rate would not be dependent on DSL for large stock. It would be DSL dependent within a range and beyond the range it is constant." The intuitive idea behind representation is as follows. First, by imposing a restriction on stock dependency of demand it's possible to obtain optimal order quantity that maximizes system profit. Second, by displaying the product above that limit maximum demand of the system can be achieved until display level drops below that level. Maiti and Maiti (2005) have also assumed the same characteristic of demand for damageable inventory and determined the optimal inventory level by using simulated annealing algorithm. Gupta et al. (2007) also developed an inventory model with three component display stock dependent demand. These models clearly indicate that above a certain level, the stock in the shelf has no impact on customers. Thus, is it necessary to display inventory in the shelf above the limit up to which customer's demand is motivated by stock in the expense of higher holding cost (often for glamorous display) rather maintaining the stock up to the level? Or, in busy market place is it beneficial to rent a storage space to stock extra inventory (inventory above the level of stock dependency) in the expense of lower holding cost?

In contrast to the studies that assume that an organization owns only a single warehouse with unlimited capacity, numerous studies are directed by focusing on two warehouses, i.e., an owned warehouse with limited capacity and a rented warehouse, which is assumed to be available with abundant capacity. This type of system has been first proposed by Hartley (1976), then several models have been discussed under various modeling assumptions by researchers like Goswami and Chaudhuri (1992), Kar et al. (2001), Zhou and Yang (2005), Dey et al. (2008), Hsieh et al. (2008), Rong et al. (2008), Chung et al. (2009), Lee and Hsu (2009) and others. In the two-warehouse inventory system it is generally assumed that holding cost in the additional storage space is higher than that of the shelf. To reduce the volume of holding cost inventory is first consumed from the storage space till it is depleted to zero. But, is it the only reason to exhaust inventory from the storage space first? In market places storage spaces are also used to stock inventory for lower holding cost than that of the shelf. In retail business generally demand is dependent on stock in the shelf. To influence customers, it is a common practice to maintain stock in the shelf. Customers' demand is filled first from the storage space till it is completely exhausted. Then it is filled from the shelf. Thus, holding cost in the storage space may be lower or higher than that of the shelf. It depends on the variety and quality of product, type of business, situation of business, place of business, etc.

In this paper we restrict our attention to instantaneous DSL dependent demand with two-component and three-component. Our work differs from the existing studies from the following aspects. First, we determine the criteria for superiority of TCD over TWCD. Second, for TCD whether a retailer would display product in the shelf or would store in rented warehouse. For this holding costs of the shelf and rented warehouse are two important parameters. We assume that holding cost of the storage space may be lower or higher than that for inventory in the shelf. Obviously it makes the model more general. Third, the two-warehouse model that is presented in the next section is more general than the traditional two-warehouse inventory model. In the traditional two warehouse inventory model the basic objective is to store additional inventory (amount of inventory that is not accommodated in the shelf) in the expanse of higher storage cost to utilize the advantage of different facilities. On the contrary in our model storage space is used irrespective of storage cost to examine the acceptability of TCD in the sense of profit maximization.

3. Mathematical modeling and analysis

To develop the model the following notations are used.

$I_{rw}(t)$	instantaneous inventory level in the rented warehouse at time t
$I_{os}(t)$	instantaneous inventory level in the shelf at time t
Q_{rs}	order quantity for the rented warehouse
Q_{os}	order quantity for the shelf
h_{rs}	holding cost per unit for the rented warehouse
h_{os}	holding cost per unit for the shelf
p	unit selling price of the product
c	unit purchase cost of the product
c_t	unit transportation cost from the rented warehouse to the shelf