Single-vendor single-buyer inventory model with discrete delivery order, random machine unavailability time and lost sales

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ABSTRACT

Integrated single-vendor single-buyer inventory model with multiple deliveries has proved to result in less inventory cost. However, many researchers assumed that the production run is perfect and there is no production delay. In reality, production delay is prevalent due to random machine unavailability and shortages. This study considers lost sales, and two kinds of machine unavailability distributions—uniformly and exponentially distributed. A classical optimization technique is used to derive an optimal solution and a numerical example is provided to illustrate the theory. The results show that delivery frequency has significant effect on the optimal total cost, and a higher lost sales cost will result in a higher delivery frequency.

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1. Introduction

Due to unreliable production system, vendors may not deliver products to the buyers when needed, resulting in lost sales. However, excessive supplies to fulfill customer's requirement results in higher inventory cost. The inventory cost is one of the dominant costs for many industries. It represents approximately 25% of the total assets (Philips and White, 1981), while the business investment on inventory is from 15–20% of the annual gross national product in the United States (Tersine, 1994). Industries should plan their strategy to provide products and services to the customers at a minimum cost. The order quantity and the time to order are critical decisions for both the manufacturing and the service industries. Some industries implement Just in Time (JIT) system to reduce their inventory cost. In order to support an efficient JIT system, it is important to ensure the reliability of the vendor's production system.

Since JIT concept can reduce inventory cost, extensive researches on vendor–buyer inventory problems with small batch deliveries have been done recently. A finite rate of production for product with lot shipment policy was initially introduced by Banerjee (1986). Goyal and Nebebe (2000) extended the model by developing a single-vendor single-buyer inventory model with small and equal sized shipment. Hoque and Goyal (2000) developed a single-vendor single-buyer integrated production-inventory

system by considering the capacity of transport equipment. A JIT model in a single-vendor single-buyer inventory system with imperfect product quality was developed by Huang (2004). Nieuwenhuyse and Vandaele (2006) proved that lot splitting policies have benefited both the supplier and the buyer. A coordinating vendor–buyer inventory model with permissible delay in payments as trade credit scenario was developed by Jaber and Osman (2006). Ertogral et al. (2007) developed an integrated vendor–buyer model under equal-size shipment and incorporated transportation cost explicitly into the model. Zhou and Wang (2007) built a single-vendor single-buyer inventory model with shortages, wherein the buyer's unit holding cost is not required to be greater than the vendor's unit holding cost and deteriorating items. Pasandideh and Niaki (2008) developed a production inventory model with multiple deliveries, multiple products and warehouse space limitation. A single-vendor single-buyer inventory model with linearly decreasing demand was developed by Omar (2009). Lin (2009) developed an integrated single-vendor single-buyer inventory model with backorder price discount and variable lead time.

All the studies above assumed that the production process is perfect and there is no delay in the production process. However in reality, there are possibilities that the production process is delayed due to machine unavailability and shortages of materials and facilities. Abboud et al. (2000) developed EPQ models by considering random machine unavailability with backorders and lost sales. The models were extended by Jaber and Abboud (2001) who assumed learning and forgetting in production. Later Chung et al. (2011) extended the work of Abboud et al. (2000) by considering deteriorating items. Some researchers have

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considered preventive maintenance time in a production inventory model (Meller and Kim, 1996; Chen, 2006; El-Ferik, 2008). The effects of machine breakdown and corrective maintenance were first studied by Groenevelt et al. (1992). Machine breakdown and corrective maintenance for a production inventory model have been extended recently by Abboud (2001), Aghessaf et al. (2007) and Chiu et al. (2008).

According to the author's extensive literature studies, there are no researches that analyze a single-vendor single-buyer (SV–SB) inventory model with JIT system and stochastic machine unavailability time. In an integrated SV–SB model, the vendor and the buyer decide jointly as a team while for a non-integrated model, the vendor and the buyer make their own decision without consulting the other. Our study on an integrated (SV–SB) model with stochastic inventory is confirmed by some researchers who have shown that an integrated SV–SB model performs better than a non-integrated model (Ben-Daya and Hariga, 2004; Lo et al., 2007).

In this study, we assume a JIT system where the buyer who pays the transportation cost, decides the order quantity size of items and requests items delivery in multiple shipments. The vendor produces the items using an economic production quantity (EPQ) model. Ideally, the machine starts a production run when the inventory level is equal to zero. In some periods, there is a possibility that the machine may not be available. If this situation occurs, the vendor cannot deliver the predetermined quantity ordered by the buyer, resulting in the buyer's lost sales. We consider two distribution models for the random machine unavailability case. The distribution models represent two different types of distribution: uniformly distributed means constant number of machine unavailability over a period of time while exponentially distributed means machine unavailability may increase with time. Both cases can occur in real life. Similar distribution types were used by Abboud et al. (2000) and Giri and Dohi (2005).

The paper has four sections. Section 1 introduces the research motivation and literature review. Section 2 shows the development of the model. Section 3 illustrates the example and sensitivity analysis. Finally, conclusions are drawn in Section 4.

2. Problem definition and formulation

2.1. Assumptions

a. A single vendor and single buyer are considered.
b. The set-up and transportation times are insignificant and can be ignored.
c. The demand rate is constant and the time horizon is infinite.
d. All costs are known and constant.
e. The buyer pays the transportation cost.
f. The unsatisfied demands of the buyer will be lost sale.

2.2. Notations

T	cycle time
T_N	total production and non production time
T_s	lost sales time
T_d	production down time
Q	the vendor's production quantity, units/cycle
q	shipment quantity, units/delivery
K	number of shipments placed during a period T_N
w	number of shipments placed during the production time
P	production rate, units/year
D	buyer's demand rate, units/year

A	buyer's ordering cost, \$/order
Av	vendor's setup production cost, \$/cycle
S_v	vendor's late delivery cost, \$/year/delivery
S_b	buyer's lost sales cost, \$/unit/year
c_t	buyer's transportation cost, \$/delivery
h_v	vendor's holding cost, \$/unit/year
h	buyer's holding cost, \$/unit/year
TBC	total buyer cost
TVC	total vendor cost
$TBUC$ ($TVUC$)	total buyer (vendor) cost per unit time
TUC	total vendor–buyer unit cost
$TBUC_{NL}$ ($TVUC_{NL}$)	total buyer (vendor) cost per unit time for no lost sales case
TUC_{NL}	total vendor–buyer unit cost for no lost sales case
$TBUC_U$ ($TVUC_U$)	total buyer (vendor) cost per unit time for uniform distribution case
TUC_U	total vendor–buyer unit cost for uniform distribution case
$TBUC_E$ ($TVUC_E$)	total buyer (vendor) cost per unit time for exponential distribution case
TUC_E	total vendor–buyer unit cost for exponential distribution case

The vendor inventory model can be seen in Fig. 1. The vendor produces products for wT_N/K time and delivers q units every shipment, where $q = Q/K$. The vendor's production quantity unit per replenishment cycle is

$$Q = wP\frac{T_N}{K} \tag{1}$$

Referring to Wang and Sarker (2006), we modify the total inventory cost to consider the case for one inventory cycle, one has

$$I_T = \frac{q^2K(K-w+1)}{2D} \tag{2}$$

The vendor's total cost consists of the vendor's setup, the holding and the shortage cost. The vendor should pay a penalty cost to the buyer when the items are delivered late. The penalty cost depends on the delivery delay time and is independent of the product quantity. The vendor's total cost in one production cycle, $T=1$, can be modeled as follows:

$$E(TVC) = Av + \frac{h_v q^2 K(K-w+1)}{2D} + S_v \int_{t=T_d}^{\infty} (t-T_d)f(t)dt \tag{3}$$

The total replenishment time consists of the production up time and production down time, and the expected shortage time.
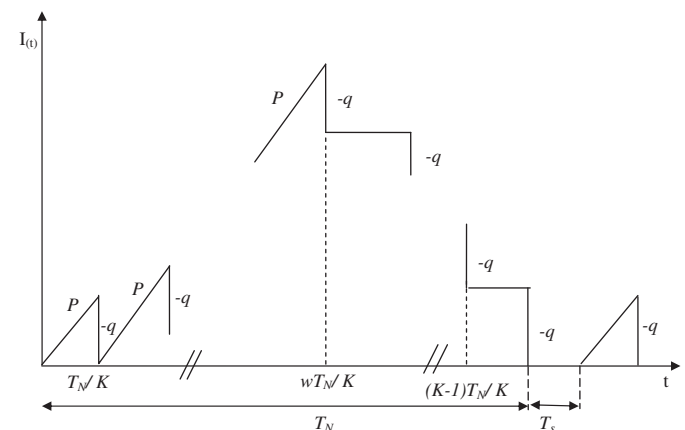

Fig. 1. The vendor inventory model.