Int. J. Production Economics 144 (2013) 180–188

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Production-inventory models for a damageable item with variable demands and inventory costs in an imperfect production process

Partha Guchhait [a,*], Manas Kumar Maiti [b,1], Manoranjan Maiti [a,2]

[a] Department of Applied Mathematics, Vidyasagar University, Midnapore 721102, Paschim-Medinipur, West Bengal, India
[b] Department of Mathematics, Mahishadal Raj College, Mahishadal 721628, Purba-Medinipur, West Bengal, India

ARTICLE INFO

Article history:
Received 9 March 2012
Accepted 29 January 2013
Available online 5 March 2013

Keywords:
Economic production quantity
Imperfect production process
Variable demand
Euler–Lagrange function
Reliability indicator
Damageable item

ABSTRACT

In this paper, economic production quantity (EPQ) models for breakable or deteriorating item are developed with variable demands, being dependent on time or on-hand stock. Here rate of production and holding cost are time dependent, unit production cost is a function of both production reliability indicator and production rate. Set-up cost is also partially production rate dependent. The production process produces some imperfect quantities which are instantly reworked at a cost to bring back those units to the perfect ones. The production process ultimately depends on both time and reliability indicator. The models are formulated as optimal control problems and the total profit functions with effect of inflation and time-value of money are expressed as finite integrals over the finite planning horizon. The problems are solved using Euler–Lagrange function based on variational calculus and Newton–Raphson method to determine the optimal production reliability indicator (r) and then corresponding production rates and total profits. In some cases, results of the models for deteriorating item are obtained as particular cases from those of breakable item models. Similarly, results of simple EPQ models (without damageability) are deduced as particular cases. Numerical experiments are performed to illustrate the models both numerically and graphically.

1. Introduction

Items available in the market can be broadly classified into two categories – damageable items and non-damageable items. Damageable items again can be classified into two sub-categories - deteriorating and breakable items. Deteriorating items are deteriorated with time and as a result holding costs of these types of items increase with time to check the increasing deterioration. Normally, seasonal goods like fruits, vegetables, X-mas cake, etc., are deteriorating in nature. Demands of these items normally exist in the market for a finite time and obviously these types of demands are dependent on time. Various types of investigation have already been made by several authors on EOQ and EPQ/EMQ models for time-dependent demand (Dave and Patel, 1981; Datta and Pal, 1991; Chen, 1998; Lee and Hsu, 2009; Sarkar et al., 2011; Maihami and Kamalabadi, 2012). Again dependency of demand on stock for most of the items in the market is well established phenomena. Some notable research papers with stock-dependent demand are Levin et al. (1972), Baker and Urban (1988), Pal et al. (1993), Hwang and Hahn (2000), Teng and Chang (2005), Soni and Shah (2008), Chang et al. (2010), Stavrulaki (2011), Zhong and Zhou (in press) and Musa and Sani (2012). Most of the above models have been formulated with infinite time horizon assuming that demand of the items exist over infinite time. According to this assumption product specification remains unchanged for ever. But in reality it is observed that unprecedental development of technology leads to rapid change in product specifications with new features. As a result, lifetimes of these types of products are normally finite and demands depend on time obviously.

On the other hand items made of glass, clay, ceramic, etc., belong to breakable category. Mainly fashionable/decorating items are made of glass, ceramic, etc., and demand of these types of items exists over finite time only. As sale of these fashionable products increases with the exhibition of stock, manufacturers of these items face a conflicting situation in their business. To stimulate the demand, they are tempted to go for huge number of production to have a large display and in this process, invites more damage to his units, as breakability increases with the increase of piled stock and the duration of stress due to the stock. Not much attention has been paid by the inventory practitioners with these types of items. According to authors' best knowledge few articles in this direction have been published by Mandal and Maiti (2000), Maiti and Maiti (2005) and Guchhait et al. (2010). Now-a-days, in the metropolitan cities, holding-cost increases with time. It increases due to increases in bank interest, etc.

* Corresponding author. Tel.: +91 9434385976.
E-mail addresses: parthaguchhait@gmail.com (P. Guchhait), manasmaiti@yahoo.co.in (M. Kumar Maiti), mmaiti2005@yahoo.co.in (M. Maiti).
[1] Tel.: +91 9434611765.
[2] Tel.: +91 9434944709.

0925-5273/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2013.02.002

Moreover, set-up cost depends partially on the production rate. Only few articles of inventory control problems have been published with variable holding cost (Alfares, 2007; Urban, 2008) and set-up cost (Matsuyama, 1995; Darwish, 2008). Till now, none has considered these assumptions in formulating the inventory models for breakable/deteriorating items.

At present, it is not possible to ignore the effect of inflation as the economy of any country changes rigorously due to high inflation. Assuming these effects on inventory costs, first impetus was given by Buzacott (1975). Among others, Beirman and Thomas (1977), Datta and Pal (1991), Ray and Chaudhuri (1997) studied some EOQ models with linear time-varying demand taking inflation and time value of money into account. Wee and Law (2004) addressed an inventory problem with finite replacement rate of deteriorating items incorporating the effect of inflation and time value of money. In the same year, Chang (2004) proposed an inventory model for deteriorating items with trade credit under inflation. In recent years, Sana (2010) and Sarkar et al. (2011) presented two inventory models in this direction.

In most of the earlier production models, inventory practitioners considered that all produced units are of perfect quality. However, in reality, all manufactured units are not always of perfect quality and directly affected by the reliability of the production process. Issue of process reliability, quality improvement and set-up time reduction have been discussed by Porteous (1986). After that, Cheng (1989) and Chung and Hou (2003) presented their models with imperfect production processes. Among others Sana et al. (2007), Yoo et al. (2009), Sarkar et al. (2010), Liao and Sheu (2011), Widyadana and Wee (2012), Dhouib et al. (2012) and Yang et al. (in press) developed different types of inventory models incorporating imperfect production process. But, none has considered inventory model with variable production rate, dynamic demand and reliability under the effect of inflation and time-value of money, specially for breakable or deteriorating item.

So in brief, the existing literature on damageable items, suffer from the following limitations.

- Most of the production-inventory models are developed for infinite time horizon (Porteous, 1986; Cheng, 1989; Maiti and Maiti, 2005; Yoo et al., 2009, etc.). According to this assumption, demand of an item remains unaltered for ever. In reality, (Gurnani, 1983) rapid development of technology leads to the change in product specifications with new features which in turn, motivates the customers to go for new product. Hence the inventory models should be developed and analyzed for a finite period of time (Khanra and Chaudhuri, 2003; Maiti, 2011, etc.). But existing literature of inventory control on damageable items overlooked this phenomenon (Maiti and Maiti, 2005; Guchhait et al., 2010). For this reason, here a finite time horizon production-inventory model of a damageable item has been formulated and solved.
- Almost all inventory models are formulated with constant or unit cost dependent holding cost (Sana, 2010; Sarkar et al., 2011; Maiti, 2011). In reality, due to inflation, bank interest, rental charges, preservation cost, etc., it increases with time. Thus some factors contributing to the holding cost change with time (Giri et al., 1996) and others remain constant. Also set-up cost depends on production rate as high production rate require sophisticated modern machineries. In this paper, holding and set-up costs are considered as functions of time and production rate, respectively.
- In imperfect production-inventory models, reliability of the production process is considered by researchers in two different approaches. In the first approach, a fraction $r(0<r\le 1)$ (called process reliability) of produced units are considered as fresh units and remaining (i.e., $(1-r)$ of the produced units) are defective units. Some authors considered r as crisp (Cheng, 1989; Maiti and Maiti, 2005) and others considered as random (Yoo et al., 2009; Liao and Sheu, 2011) and tried to determine optimal r to optimize cost or profit. In their considerations production may be 100% perfect by introducing high quality machineries, which is unrealistic in most of the real life production systems. In reality manufactures are highly satisfied if r reaches a maximum level and they never allow r to fall below a minimum level. Considering this phenomenon, in second approach, recently some works have been done by Sana (2010), Sarkar et al. (2011) and Sarkar (2012). Due to feasibility, in the present investigation, the second approach is considered.
- It is a common business practice that customers are allured with displayed stock and for that, demand is considered as stock-dependent (Levin et al., 1972; Baker and Urban, 1988; Alfares, 2007; Stavrulaki, 2011, etc.). But, in the case of breakable items, manufacturer cum retailers are in dilemma to have large displayed stock as increased stock invites more breakability along with more sale. Hence, a balance is to be maintained between increased breakability and sale for maximum profit. Till now, little attention has been paid by the inventory practitioners to consider this phenomena for a production-inventory problem of damageable items (Maiti and Maiti, 2005). Here optimum reliability indicator and then inventory level for breakable items (such as units made of china-clay, mud, ceramic, etc.) keeping a balance between increased sale and wastage are found to have maximum profit.
- Variational principle is a simple technique for the analysis of optimal control problems. Very few researchers have developed the production-inventory models as optimal control problems and solved using this method (Sarkar et al., 2011). The present problem has been solved by this method.
- Moreover, in most of the above models, unit production cost is assumed to be constant. In practice, it varies with production rate, raw material cost, labour charge, wear and tear cost and reliability of the production process (Khouja, 1995). In this investigation, unit production cost is dependent on production rate, reliability indicator, raw material, labour charge and wear-and-tear costs.
- Till now, no research work is reported taking the above features into account in a single model. Sana (2010) recently presented an imperfect production-inventory model for a non-breakable item with three types of demands taking some of the above assumptions into consideration along with constant holding. As mentioned earlier, units made of china-clay, ceramic, etc., have a special property of getting broken due to stress and this breakability increases with time. Here for the first time, a production-inventory model is formulated and analyzed for such materials with general form of breakability and variable inventory costs. Optimum results have been derived with three types of demand, of which one increases with time at a decreasing rate, not considered by others.

In this paper, a production-inventory model with imperfect production process is considered for a breakable or deteriorating (as a particular case) item over a finite time horizon. The production rate varies with time. Here different models have been formulated with different types of demand function i.e., time dependent or stock-dependent. Set-up cost is partially production rate dependent and holding cost is also partially time dependent. The unit production cost is a function of production rate, raw material cost, labour charge, wear and tear cost and product reliability indicator. The models are formulated as optimal control problems for the maximization of total profits over the planning horizon and optimum