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A vendor–buyer integrated production–inventory model with normal distribution of lead time

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ABSTRACT

By relaxing the long-term assumption of the deterministic lead time, recently three coordinated vendor–buyer models with exponential distribution of lead time in a two-stage supply chain were presented. The vendor produces a product at a finite rate and delivers the lot to the buyer with a number of equal-sized batches (sub-lots) to meet the deterministic demand. The next batch is ordered when the previous one drops to a reorder point. Shortages were allowed and completely backordered. However, in exponential distribution of lead time, the probability of arrival of a batch earlier is higher than the probability of arrival of a batch late or in the mean lead time. But usually, probability of arrival of a batch earlier or late appears to be smaller than the probability of arrival of a batch in the mean lead time. Thus normal distribution of lead time seems to be a better fit to the problem. Hence their models seem unfit to the concerned problem in practice. Based on this notion, we develop a vendor–buyer integrated production–inventory model following normal distribution of lead time but retaining their other assumptions. To make the model more realistic, set up time per set up of a machine, the highest limit on the capacity of the transport vehicle and the transportation cost and time per batch are imposed. Then we derive an optimal solution technique to the model to obtain minimum expected joint total cost that follows development of the solution algorithm. Extensive comparative studies on the results of some numerical problems are carried out to highlight the potential significance of the present method. Sensitivity analysis to the solutions with variations of some parameter values are also carried out.

1. Introduction

This paper deals with the development of a vendor–buyer integrated production–inventory model with normal distribution of the delivery lead time of a batch, and its optimal solution technique. It has been agreed by the researchers that the integrated inventory system plays an important role for efficient and effective management of inventories across the entire supply chain. Vendor–buyer coordination is essential for successful implementation of an integrated inventory model (e.g., Goyal and Guptha, 1989; Chandra and Fisher, 1994; Thomas and Griffin, 1996; Sharma et al., 2006; Tarantilis, 2008). Integrated inventory models have been developed including various concerned factors of the system (see for detail Sajadieh et al., 2009) to enrich the literature. One of the important factors is the delivery lead time of a batch. Although most of the models have been developed based on the deterministic lead time, researchers have enriched the literature with the development of models by taking into account controllable lead time with extra cost (e.g., Ben-Daya and Rouf, 1994; Ouyang et al., 1996; Ouyang and Wu, 1997, 1998; Moon and Choi, 1989; Lan et al., 1999; Pan and Hsiao, 2001; Pan et al., 2002; Pan and Yang, 2002; Chang, 2005; Pan and Hsiao, 2005; Hoque and Goyal, 2006; Chang et al., 2006; Hoque, 2009; Ye and Xu, 2010). Inventory policies have also been developed with stochastic lead time (e.g., Yano, 1987; Kumar, 1989; Fujiwara and Sedarage, 1997; Rossi et al., 2010). Glock (2012) developed an integrated inventory model for the single-vendor single-buyer with stochastic demand and variable lead time under different lead time reduction strategies. He adopted the formulation of the lead time from Hsiao (2008). The lead time of the first batch is formulated by considering production time and setup and transportation time while the lead time for 2,3,…,n batch is only the transportation time considered in the model. However, a little attention has been given for developing a vendor–buyer integrated production–inventory model with stochastic lead time. Recently Sajadieh et al. (2009) developed such a model with exponential distribution of lead time and allowing backordering of shortages.

They developed three models: two of them considering total cost of the vendor and the buyer individually and the third for the

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integrated system, for delivering a lot of a product (produced by the vendor) to the buyer in n number of equal-sized batches to meet the deterministic demand. An order is placed by the buyer when the stock level falls to a certain level, called the reorder point. They identified some environmental causes such as order processing and transportation times of a batch, inspection etc. as factors of leading to lead time uncertainty. Being motivated by this idea they developed the models with exponential distribution of lead time and allowing backordering of shortages. The purpose was to show significant reduction in the minimal total cost of ordering, set up, inventory holding and shortages by the integrated system. In the exponential distribution, the probability of a variable less than of its mean value is always higher than the probability of a variable greater than or equal to its mean value. In case of the exponential distribution of lead time, if a batch arrives early, then its lead time is less than the mean lead time and if a batch arrives late, its lead time is greater than the mean lead time. So, the probability of earlier arrival of a batch is always higher than the probability of arrival of a batch late or in the mean lead time in this distribution, application of which in their models seems to be impractical. To meet the demand without allowing planned shortages, the set up time of a machine plus the processing time of the first batch of a next lot plus its loading, unloading, transfer and inspection time (t) must equal to the time of meeting the demand by the units at the reorder point. This constraint has not been taken into consideration in their models, and hence it may lead to planned early arrivals or late deliveries always. To overcome this deficiency we have taken into account this constraint in developing the model in this paper. Besides, sometimes the processing time of the first batch plus t may deviate because of various factors. Sometimes, other batches may arrive earlier or late because of variations in t. Thus, in practice, the probability of arrival earlier or late of a batch seems to be smaller than the probability of arrival of a batch in the mean lead time, and usually the former probabilities appear to be more or less symmetrical to the latter ones. Thus the lead time seems to follow the normal distribution. For this reason, for a known value of t, here we develop a vendor–buyer integrated production–inventory model following the normal distribution of the lead time but retaining Sajadieh et al. (2009) other assumptions. Also, when a batch arrives late to the buyer, it is kept elsewhere for the same amount of time of delay and creates an extra inventory there. If the late delivery is due to late start of production of a product, then its raw material creates an extra inventory. Sajadieh et al. (2009) did not take into account this extra inventory in developing their models. But this is taken into account in developing the present model here. In addition, we have considered the highest capacity of the transport vehicle used to transport the product because it may not be unlimited. Moreover, the set up time per set up of a machine, the transportation cost and time for transporting a batch to the buyer are also imposed. Thus we have developed the model with the assumption of the normal distribution of the lead time (but retaining their other assumptions), and taking into account the mentioned extra inventory and the constraints along with the set up time and the transportation cost and time per batch. Then a number of properties leading to the minimal total cost (of set up, ordering, inventory holding, transportation and shortage) to the model are established, and hence the optimal solution algorithm is obtained. Extensive comparative studies on the results of numerical problems are carried out to highlight the potential significance of the present method. Sensitivity analysis to the solutions found with variations of some parameter values are also carried out.

We organize the paper as follows: In the next section we put forward assumptions and notations. Section 3 deals with development of the model with the normal distribution of the lead time, and its minimal total cost solution technique. In Section 4 we carry out extensive comparative studies on the results of numerical problems. Section 5 concludes by highlighting the paper findings, limitations and future research directions.

2. Assumptions and notations

The assumptions in developing the model are as follows:

i) Deterministic constant demand and production rates over an infinite time horizon, where the latter is greater than the former;
ii) A product is produced by the vendor and the lot is transferred to the buyer with a number of equal sized batches;
iii) Shortages are allowed and completely backordered;
iv) The unit time shortage cost is higher than the unit time product holding cost at the buyer (Fujiwara and Sedarage, 1997);
v) Lead time to replenish the buyer's order is variable and follows the normal distribution;
vi) Inventory at the buyer is continuously reviewed, and a batch is ordered when the inventory drops to a certain reorder point;
vii) Both the vendor and the buyers agree to share the benefit of integrated inventory system through negotiation but without incurring any cost;
viii) The orders do not cross in time (Sajadieh et al., 2009).

We use the following notations in developing the model:

D Demand rate; P production rate ($P>D$ and $k=P/D$);
Q Batch size; n Number of batches in a lot; L Random lead time to replenish the buyer's order;
n Total number of equal sized batches in a lot; r Buyer's reorder point;
A_v Vendor's set up cost per set up of a lot; A_b Buyer's ordering cost/order;
g Highest transport capacity of the transport vehicle; T Transportation cost/batch.
h_v Inventory holding cost/unit/unit time for the vendor;
h_b Inventory holding cost/unit/unit time for the buyer ($h_b>h_v$);
π' Shortage cost/unit/unit time for the buyer (($h_b<\pi'$); s set up time per lot;
t Loading, unloading, inspection and transportation time of a batch;
σ Standard deviation of lead time.

3. Formulation of the model and development of its optimal total cost solution technique

3.1. The model

3.1.1. The joint total cost function

In this integrated production–inventory system, the vendor produces a lot of a product by a single set up following the receipt of its order from the buyer. Based on an agreement between the two parties, the lot is transferred by the vendor in n number of equal-sized batches of size Q. However, each of them is transferred to the buyer when it is ordered by him/her. The buyer places an order of a next batch when the current one at his/her