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Retail inventory management with stock-out based dynamic demand substitution

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ABSTRACT

We consider an inventory management problem of a product category in a retail setting with Poisson arrival processes, stock-out based dynamic demand substitution, and lost sales. The retailer uses a fixed-review period, order-up-to level system to control the inventory levels. We present a computational method to determine the order-up-to levels that maximizes the expected profit with profit margins, inventory holding and substitution costs subject to service-level constraints. Determining expected sales, average inventory levels, and number of substitutions between all products for given demand rates, substitution probabilities, and order-up-to levels is not tractable when there are more than two products. Therefore we present efficient and accurate approximations to approximately compute the same performance measures. The approximate approaches are then used to solve the optimization problem by using a genetic algorithm. In a computational study, we discuss the impact of profit margins, inventory holding and substitution costs, and service level constraints on the order-up-to levels and the expected profits. We show that a retailer can increase its expected profits by incorporating substitution among different products.

1. Introduction

This paper studies an inventory management problem in a retail setting with stock-out based substitutions and multiple items in a product category and proposes an approximate solution to determine the order-up-to levels to maximize the expected profit subject to service level constraints. The method uses demand parameters including the substitution probabilities estimated from the point-of-sales data. As a result, the method provides a practical tool for retailers to manage their inventory.

The literature on inventory management under stock-out based substitutions studies the supplier-(or manufacturer-)controlled and customer-driven substitution schemes. In the supplier-controlled substitution scheme, in a stock-out instance, the supplier decides whether to fulfill the demand of the customer with another product. The inventory management (and/or production planning) problem is usually studied in a "one-way substitution" setting, where a higher-graded product can be substituted for a lower-graded product. The primary objective is to minimize the sum of production, inventory holding, and, in some cases, product conversions costs. A detailed discussion of the relevant literature on supplier-controlled substitution is presented in Hsu et al. (2005) and Rao et al. (2004).

In this paper, inventory management under the customer-driven substitution scheme is studied. In the customer-driven substitution scheme, when the first-choice product of the customer is not available on the shelf, the customer may purchase, with a certain probability, another product in the same category in lieu of her first-choice product. Although the retailer can only indirectly affect customers' decisions through his inventory management decisions, ignoring product substitutions in managing the inventories may result in sub-optimal performance: Mahajan and van Ryzin (2001) analyze a single-period, stochastic inventory problem with substitutable products, and show that "substitution effects can have a significant impact on an assortment's gross profits." Ernst and Kamrad (2006) study a two-product problem with customer-driven substitution in a newsvendor setting, and conclude that "using a Newsboy Model framework without regard to substitutions can be sub-optimal."

In the single-period models, it is usually assumed that the demand realizes at the end of the period. A single-product problem can be analyzed under this assumption; however in a multi-product/customer-driven substitution setting, the dynamics of the problem is quite different because of the customer arrival process. Smith and Agrawal (2000) and Mahajan and van Ryzin (2001) present models, with underage and overage costs, that account for customers' arrival order in finding the optimal order quantities. Hopp and Xu (2008) approximate the dynamic substitution behavior with a fluid network model, and study inventory, price, and assortment decisions in centralized and decentralized settings.

The inventory management problem with static substitutions has been extensively studied in the literature. McGillivray and Silver (1978) study a periodic review system with substitutable items having the same unit variable cost and shortage penalty,

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and develop an upper bound on the inventory and shortage costs savings that could be achieved when the product substitution is taken into account in choosing the order-up-to-levels. Parlar (1985) generalizes the newsvendor problem with a product that perishes in two periods, and assumes that the one-period-old and fresh products are substitutable. Parlar (1985) presents an infinite horizon Markov decision model to find the optimal ordering policy. Avsar and Baykal-Gursoy (2002) analyze the competition of two retailers that offer substitutable products, and present a two-person stochastic game to characterize the Nash equilibrium. Rajaram and Tang (2001) study a multi-product newsvendor problem with substitutability, and analyze the impact of demand uncertainty on order quantities and expected profits. Netessine and Rudi (2003) study a single-period problem where unsatisfied demand for a product flows to other products in deterministic proportions, and present analytically tractable solutions for comparing the profits of the centralized and competitive inventory management settings. Nagarajan and Rajagopalan (2008) study a two-product problem with negatively correlated demands. The substitution proportions from the first to the second and from the second to the first product are assumed to be identical. Nagarajan and Rajagopalan (2008) first show that, in a single-period setting and when the substitution proportion is not very large, the optimal base-stock levels are not state-dependent. In a computational study, they also show that a heuristic based on the solution of the two-product problem performs well with multiple products and under general conditions.

A closely related research stream studies customer-driven substitution in the context of assortment planning. Kök and Fisher (2007) study an assortment planning model with substitutable products, develop a procedure for estimating substitution parameters, and present a heuristic for solving the assortment planning problem. Yücel et al. (2009) study assortment and inventory planning problems under customer-driven substitution in retail operations. They show that ignoring substitutability of products or shelf space limitations may result in sub-optimal assortments. Detailed reviews of the literature on assortment planning have been presented by Kök et al. (2008) and Mahajan and van Ryzin (1998).

Following up on our earlier work on the estimation of substitution probabilities (Karabatı et al., 2009), the objective of this paper is to develop an easily implementable method to determine the optimal order-up-to levels that maximize the expected profit of the system. The inventory management method we propose incorporates the effects of stock-out based dynamic substitutions. The method we developed attempts to answer the question how much the total profit of a product category can be increased by setting the order-up-to levels in a way that captures the effects of substitution and profitability of the products. For example, an inventory plan may force customers of products with low profit margins to substitute with higher profit margin products by setting the order-up-to levels intentionally low to cause stock outs. Our first attempt to analyze this problem is given in the thesis of Helvacıoğlu (2009) where different approximation methods are used together with mathematical optimization techniques.

Our paper contributes to the literature discussed above by considering a multi-period multi-product problem that incorporates stock-out based dynamic substitutions, substitution costs, and service level requirements. Combining with the method developed to estimate the substitution probabilities, the proposed method works directly with the point-of-sales data and suggests order-up-to-levels to the retailers.

This paper is organized as follows. Section 2 provides a description of the problem. In Section 3, an exact analysis of inventory system's performance for the two-product case is presented. Section 4 presents deterministic and probabilistic approaches to approximately compute the performance measures of interest. Section 5 provides a computational analysis of the approximation approaches. The optimization of the order-up-to levels under stock-out based dynamic substitution is investigated with numerical results in Section 6. Section 7 concludes the paper.

2. Problem description

We consider a retailer that stocks and sells N products in a category. Demand for Product i is a Poisson random variable with rate λ_i, $i=1,\ldots,N$. If a customer, whose first-choice product is Product i, cannot find it on the shelf, she may substitute it with Product j with probability α_{ij}. The substitutions probabilities, which can be estimated with methods discussed in Anupindi et al. (1998) and Karabatı et al. (2009), are an input of our problem. We assume that the customers make only one substitution attempt, and the demand is lost if their second-choice product is not available either. Kök et al. (2008) state that it is possible to approximate a multiple-substitution attempt model with a single-attempt model by adjusting the parameters. Furthermore when service levels are reasonably high, most customers find their first- or second-choice product on the shelf, eliminating the possibility of a second substitution attempt.

The retailer uses a fixed review period, order-up-to level system to control the inventory. The review period is equal to T time units, and the order-up-to level for Product i is Q_i, $i=1,\ldots,N$. The demand of Product i during the review period is denoted by D_i, and is a Poisson random variable with rate $\lambda_i T$.

2.1. Performance measures

The performance measures we are interested in are the expected sales (total, direct, and through substitution) of products, the expected service and inventory levels, and system's expected profit during a review cycle:

Expected sales: The expected total sales of Product i during a review cycle is denoted by $S_i, i=1,\ldots,N$. The expected number of units of Product i sold to the customers of Product j during a review cycle, who substituted Product j with Product i due to the unavailability of Product j, is denoted by S_{ji}, $j,i=1,\ldots,N; j\neq i$. Therefore, the expected number of substitution sales of Product i during a review cycle is equal to $\sum_{j\neq i} S_{ji}$. The expected number of units of Product i sold during a review cycle to the customers of Product i, i.e., direct sales of Product i, denoted by S_{ii}, $i=1,\ldots,N$, is then equal to $S_i - \sum_{j\neq i} S_{ji}$.

Service levels: In a multi-item retail setting with dynamic demand substitution, service levels can be measured in different ways. Measuring service level as the ratio of expected sales to expected demand is harder to interpret, since this measure can go over 100% as a result of substitutions. In addition, this measure does not differentiate between sales to customers who buy these products as their first choices and sale to customers who buy these products as a result of substitution due to not finding their first choice in stock. Accordingly, we use a service level measure, SL_i, $i=1,\ldots,N$, that is defined as the ratio of total direct sales of Product i to the total demand of Product i during a review cycle:

$$SL_i = \frac{S_{ii}}{\lambda_i T}. \tag{1}$$

Inventory level: The average inventory level for product i is denoted by $\bar{I}_i$, $i=1,\ldots,N$.