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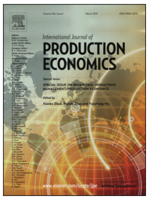

Optimal lot size for a production–inventory system with partial backlogging and mixture of dispatching policies

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ABSTRACT

In this paper we study a production–inventory model for a single product where shortages are partially backlogged. Both the backorder cost and the lost sale cost depend on a fixed cost and a cost proportional to the shortage time. We assume a mixture between the dispatching policies known as LIFO (last in, first out) and FIFO (first in, first out) in the discipline of service to fill the backlogged demand. By using a sequential optimization procedure, we determine in a closed-form the optimal production policies and the optimal profit for all the possible cases obtained from the developed model. We show how this new model generalizes several inventory models proposed in the literature. Finally, numerical results are presented to analyze the sensitivity of the optimal policies with respect to changes in some parameters of the system.

1. Introduction

The acquisition, production and distribution of goods for inventories are issues of concern to all organizations. Very large costs are usually incurred as a result of replenishment or production actions, shortages and utilization of managerial and clerical time in making and routinely implementing inventory management decisions. Thus, properly designed decision rules, based on mathematical modeling, can lead to substantial benefits.

The economic order quantity (EOQ) model proposed by Harris (1913) and subsequently popularized by Wilson (1934) was the initial lot size model based on cost minimization (see, e.g. Lee and Nahmias, 1993, p. 4). An early extension of the basic EOQ model, generally referred as Economic Production Quantity (EPQ) or Economic Manufacturing Quantity (EMQ) model, replaced the assumption of instantaneous replenishment by the assumption that the replenishment is scheduled at a constant finite rate over time (see Taft, 1918).

The fundamental purpose of an inventory control system consists of solving the following two issues or problems: (i) when a replenishment order should be placed and (ii) how large the replenishment order should be. To answer these two points, inventory models have to be developed. The assumptions made about the parameters of the system, the structure of the cost functions and the physical characteristics of the system play an important role in determining the complexity of the resulting models. There are a great number of variations in inventory models, each suitable for a specific situation.

Many extensions of the basic EOQ and EPQ models have been constructed and solved through the formulation of different assumptions. An excellent analysis of more than three hundred inventory models is provided by Chikán (1990) and the literature on this topic is continually increasing. Besides, in the few last years, new works on the classical models have appeared. Some of these papers study the methodology used in the teaching of the inventory models. This approach is followed, among others, by Ronald et al. (2004), Chang et al. (2005), Sphicas (2006), Minner (2007), Leung (2008), Cárdenas-Barrón (2009, 2010, 2011), Chang and Ho (2010), Hsieh and Dye (2012), Chen et al. (2013) and Ouyang and Chang (2013), where the optimal lot size formulae for the EOQ and EPQ models are developed by using basic geometric and algebraic methods.

As it is very well known, in Inventory Control one of the most important situations takes place when a customer orders an article which is not available. In such a framework two issues are usually considered in the literature on this topic: either the demand during the stockout period is completely backordered (full backorder case), or the total demand during the stockout period is lost, because the customer leaves the system (lost sales case). However, many practical situations include a combination of these extreme issues. That is, taking into account the interests or necessities of

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the customers, a fraction β of the demand in every shortage period corresponds to customers who are willing to wait until the arrival of a new lot and the rest are lost sales. Despite the fact that this mixed situation has sometimes been considered approachable from either of the extreme issues (Silver et al., 1998, p. 234), some authors thought differently. Examples of this can be found in Montgomery et al. (1973), Rosenberg (1979), Park (1982), Yan and Cheng (1998), Wee (1999), Goyal and Giri (2003), Chu and Chung (2004), Yang (2007), Leung (2009), Pentico and Drake (2009), San-José et al. (2009a, 2009b), Zhang et al. (2011), Taleizadeh et al. (2012, 2013a, 2013b), Karimi-Nasab and Konstantaras (2013) and Taleizadeh and Pentico (2013). More specifically, these researchers consider that only a fixed fraction β of the demand during the shortage period is served late. In addition, Mak (1987), Pentico et al. (2009, 2011), Drake et al. (2011), Toews et al. (2011), Wee and Wang (2012), Chung (2013) and Stojkovska (2013) studied EPQ systems considering partial backlogging and uniform rate of production, which represent the replacement capacity per unit of time. They developed several approaches to determine the economic production quantity. We refer to Pentico and Drake (2011) for a survey of deterministic models for the EOQ and EPQ with partial backordering.

As has already been shown in other research lines within Inventory Control (see, for instance, Cohen and Prastacos, 1981; Lee, 2006; Niu and Xie, 2008; Parlar et al., 2011), in the EPQ models with partial backlogging, it is also necessary to know the discipline of service to fill the backlogged demand in the system. Assuming that none of the backorders will convert to lost sales, an important question is the following: What happens with the demands that are arriving in the system when there is no stock on hand, but the production has started? In general, there are two extreme ways to answer this question. If we assume that incoming demands are filled from production before backorders (LIFO dispatching policy), then the net inventory level will be as shown in Fig. 1 (dotted line with slope $P-D$). This is so because in the production–inventory system, the input rate is P and the output rate is D. However, if we assume that the backorders will be filled before the new demands (FIFO dispatching policy) and, moreover, we admit that only a fraction β of these new demands will be backordered, with the rest being lost sales, then the net inventory level will be as is shown in Fig. 1 (dotted line with slope $P-\beta D$). In this case, the input rate is also P, while the output rate is βD. Note that if all shortages are completely backordered (that is, $\beta=1$), then there is no difference between LIFO or FIFO dispatching policies (see dotted line with slope $P-D$ in Fig. 1). Based on shortage type, backorder cost, lost sale cost, solution method and dispatching policy, some EPQ inventory models are summarized in Table 1.

In this paper we study a production and inventory system with partial backlogging where a fraction of customers have priority in the service. This situation occurs frequently in business and enterprises, because in the commercial activity there are some customers who are very important people and have preference with respect to the rest of customers. We would like to comment that the system here proposed is different from the existing EPQ models in various aspects. Firstly, we consider that both backorder unit cost and lost sale cost are made up of a fixed cost and a variable cost which depends on the length of the waiting time up to the next production. Thus, four parameters are considered to determine the shortage cost. If we suppose that some of these parameters are zero, different inventory models are obtained. Secondly, in the shortage period we assume that the discipline in the service used through the system is a mixture between the LIFO and FIFO dispatching policies, because we suppose that a part of the customers have priority to be firstly served when the production is running. That is, if there is no stock on hand but the production has started, a part γ of the incoming demands will be filled from production before backorders are attended. With respect to the other part (that is, the $1-\gamma$ fraction of the incoming customers) a fraction β are willing to wait until the next replenishment and the rest leave the system. The customers who are willing to wait are filled after the previous backorders are satisfied. Obviously, if $\gamma=1$ we revert to the LIFO dispatching policy and when $\gamma=0$ we obtain the FIFO approach.

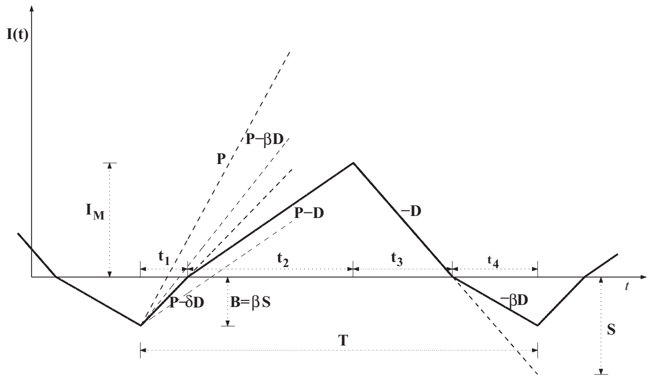

Fig. 1. Net stock level on an inventory cycle.

In consequence, this system becomes more general than other models which have previously been proposed in the literature, as can be easily tested assuming particular values for some of the parameters here considered. In particular, this new model includes those EPQ systems studied in Mak (1987), Tersine et al. (1992), Tersine (1994), Pentico et al. (2009) and Cárdenas-Barrón (2011). Furthermore, if we consider that the production rate is infinite we obtain, among others, the EOQ systems studied by Hadley and Whitin (1963), Montgomery et al. (1973), Rosenberg (1979), Park (1982), Chu and Chung (2004), Yang (2007), Leung (2008, 2009) and San-José et al. (2009a).

This paper is organized as follows. Section 2 presents the assumptions and notation used throughout the paper. Section 3 deals with the formulation of the model, whose mathematical analysis is given in Section 4. Section 5 provides some particular cases to illustrate the model here studied. Numerical examples are given in Section 6. The conclusions are set up in Section 7. The paper ends with an appendix, where some minor results are proved.

2. Assumptions and notation

We consider the following assumptions:

- The production and inventory system is based on a single item.
- The system operates over an infinite planning horizon.
- The inventory system has uniform demand pattern and the demand rate is known and constant.
- The inventory is continuously reviewed.
- The replenishment is continuous with a known and finite rate.
- The lot size per cycle is constant.
- The setup cost is fixed regardless of the lot size.
- The holding cost is a linear function of the average inventory.
- The system allows shortages which are partially backlogged. If a customer arrives in the system and he/she is not immediately served, then the customer can wait or leave the system. That is, there exist impatient customers who do not want or cannot wait for the next replenishment and other customers who are willing to wait to satisfy their demand. It is considered that a fixed fraction of demand is backlogged.
- The cost of a backorder includes a fixed cost and a cost which is proportional to the length of time for which backorder exists.

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