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Optimizing replenishment policy in an EPQ-based inventory model with nonconforming items and breakdown

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ABSTRACT

The optimal replenishment policy for an economic production quantity (EPQ)-based inventory model with nonconforming items and breakdown is presented. A real-life production system inevitably generates nonconforming items and has equipment breakdowns owing to process deterioration or other uncontrollable factors. This study addressed these issues in an EPQ-based system to optimize a replenishment policy that minimizes the long-run average cost for the proposed system. Whenever a breakdown occurs, the machine is assumed to immediately be under repair, and an abort/resume inventory control policy is adopted. Under this control policy production of the interrupted lot resumes immediately after the machine is fixed and restored. A mathematical model and a recursive algorithm were used to derive the optimal replenishment policy. A numerical example was used to demonstrate the practical application and better cost efficiency of the proposed policy compared to a breakdown that occurs under a no-resumption policy.

1. Introduction

This paper is concerned with determining the optimal replenishment policy for an economic production quantity (EPQ) based inventory model with nonconforming items and random machine breakdown. Harris (1913) first introduced the economic order quantity (EOQ) model to assist corporations in minimizing total inventory costs. He employed the mathematical techniques to balance the setup and stock holding costs in order to derive the optimal order size that minimizes the long-run average cost. For the manufacturing firms, when items are produced in-house instead of being acquired from outside suppliers, the EPQ model is often adopted to cope with the non-instantaneous stock replenishment rate in order to obtain minimum production-inventory cost per unit time (Taft, 1918). Disregarding the simplicity of the original EOQ and EPQ models, the concept of cost minimization and the technique of mathematical modeling remain broadly used (Nahmias, 2009; Silver et al., 1998). Quite a few more complicated and practical production-inventory models have since been extensively studied and developed (Alghalith, 2013; Bylka, 2003; Chen et al., 2012; Chiu et al., 2013; de Kok, 1985; Hadley and Whitin, 1963; Kohli and Park, 1994; Latha Shankar et al., 2013; Mishra et al., 2011; Sana, 2012; Schneider, 1979).

The classic EPQ model implicitly assumes that all items made are of perfect quality. However, in real-life manufacturing systems, due to process deterioration or various other factors, production of imperfect quality items is inevitable. Studies that extended the EOQ and EPQ models by undertaking issues of the defectiveness and its corresponding quality cost have been broadly conducted (Chen et al., 2013; Chiu et al., 2010, 2011a; Hariga and Ben-Daya, 1998; Mahata, 2012; Pal et al., 2012; Rahim and Ben-Daya, 2001; Rosenblatt and Lee, 1986; Sarkar and Sarkar, 2013). Samples of articles are surveyed as follows. Hariga and Ben-Daya (1998) studied the economic production quantity problem in the presence of imperfect processes. The time to shift from the in-control state to the out-of-control state was assumed to be flexible, and they provided distribution-based and distribution-free bounds on the optimal cost respectively. For the exponential case, they compared the optimal solutions to approximate solutions proposed in the literature. Rahim and Ben-Daya (2001) examined the simultaneous effects of both deteriorating product items and deteriorating production processes on the economic production quantity, inspection schedules, and the economic design of control charts. Deterioration times for both product and process were assumed to follow arbitrary distributions, and the product quality characteristic was assumed to be normally distributed. Applications of their models were demonstrated through illustrative examples. Chiu et al. (2011a) studied an economic manufacturing quantity (EMQ) with rework and multiple shipments. They incorporated quality assurance and multiple deliveries into classic EMQ model and derived the optimal inventory

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replenishment policy that minimizes the expected total production-inventory costs. Two special cases to their proposed model were discussed and examined.

Unexpected breakdown of the production equipment is another critical reliability factor which can be disruptive when occurring — especially in a highly automated production environment. Groenevelt et al. (1992) studied two control policies that deal with random machine breakdown. The first one assumes that after a breakdown the production of the interrupted lot is not resumed (called the no resumption-NR policy). The second policy considers that the production of the interrupted lot will be immediately resumed (called the abort-resume-AR policy) after the breakdown is fixed and if the current on-hand inventory is below a certain threshold level. The repair time is assumed to be negligible in their study. The effect of machine breakdown and corrective maintenance on the economic lot size decisions is investigated. Studies have since been carried out to address the issue of machine failures during production (see for instance, Abboud, 2001; Arreola-Risa and DeCroix, 1998; Berg et al., 1994; Chakraborty et al., 2009; Chiu et al., 2011b, 2012; Das et al., 2011; Giri and Dohi, 2005; Makis and Fung, 1998; Moinzadeh and Aggarwal, 1997). Moinzadeh and Aggarwal (1997) studied a production-inventory system that is subject to random disruptions. They assumed that the time between breakdowns is exponential, restoration times are constant, and excess demand is backordered. An (s, S) policy was proposed and the policy parameters that minimize the expected total cost per unit time were investigated. A procedure for finding the optimal values of the policy was also developed. Arreola-Risa and DeCroix (1998) explored an (s, S) stochastic-demand inventory management system under random supply disruptions and partial backorders. Their analysis yields the optimal values of the policy parameters and provides insight into the optimal inventory strategy when there are changes in the severity of supply disruptions or in the behavior of unfilled demands. Giri and Dohi (2005) developed an exact formulation of stochastic EMQ model for an unreliable production system. Their model is formulated based on the net present value (NPV) approach, and by taking limitation on the discount rate the traditional long-run average cost model is obtained. They also provided the criteria for the existence and uniqueness of the optimal production time and computational results showing that the optimal decision based on the NPV approach is superior to that based on the long-run average cost approach. Chakraborty et al. (2009) investigated the lot size problem with process deterioration and machine breakdown under inspection schedule. Chiu et al. (2011b) studied the manufacturing run time problem with random defective rate and stochastic machine breakdown under a no resumption (NR) inventory control policy. Modeling and numerical analyses were used in order to establish the solution procedure. As a result, the optimal run time that minimizes the long-run average production-inventory cost is derived.

For the reason that little attention was paid to the investigation of the joint effects of random nonconforming rate and machine breakdown (under the AR inventory control policy) on the optimal replenishment policy of the EPQ-based system, this paper intends to bridge the gap.

2. Problem description

Consider a production system has a random defective rate x. All nonconforming items produced cannot be repaired they are scrap. Further, the system is subject to a machine failure and it may take place randomly. When a breakdown occurs, the abort/resume (AR) inventory control policy is adopted. Under such an AR policy, the malfunction machine is immediately under repair and the interrupted lot will be resumed right after the restoration of machine. The repair time is assumed to be constant. There is a safety stock policy in place to cope with the unwanted potential shortage situation when the machine breakdown occurs in very earlier stage of the production.

The production rate is constant and is much larger than the demand rate λ. The production rate of defective items d can be expressed as the production rate times the defective rate: $d = Px$. The cost parameters considered include the setup cost K, unit production cost C, unit holding cost h, disposal cost per scrap item C_S, and a fixed cost M for repairing and restoring the machine. Additional notation is listed as follows.

t_1	the production uptime to be determined for the proposed model,
β	number of breakdowns per year, a random variable follows a Poisson distribution,
t	time before a random breakdown occurs,
t_r	time required for repairing the machine,
t_2'	time needed for consuming all available good items when breakdown takes place,
H_1	the level of on-hand inventory when machine breakdown occurs,
H_2	the level of on-hand inventory when machine is repaired and restored,
H_3	the level of on-hand inventory when machine is restored and the remaining production uptime is accomplished,
Q	production lot size per cycle,
$I(t)$	on-hand inventory of perfect quality items at time t,
$I_d(t)$	on-hand inventory of defective items at time t,
T'	cycle length in the case of machine breakdown takes place,
$TC_1(t_1)$	total inventory costs per cycle in the case of machine breakdown takes place,
t_2	time required for depleting all available perfect quality items when machine breakdown does not occur,
H_4	maximum level of on-hand inventory in units when production process ends,
T	cycle length when machine breakdown does not occur,
$TC_2(t_1)$	the total inventory costs per cycle when machine breakdown does not occur,
$\mathbf{T}$	cycle length whether a machine breakdown or not,
$TCU(t_1)$	the total inventory costs per unit time whether a breakdown takes place or not.

In order to accumulate a positive level of stock, one fundamental assumption for the proposed EPQ model is that the production rate of

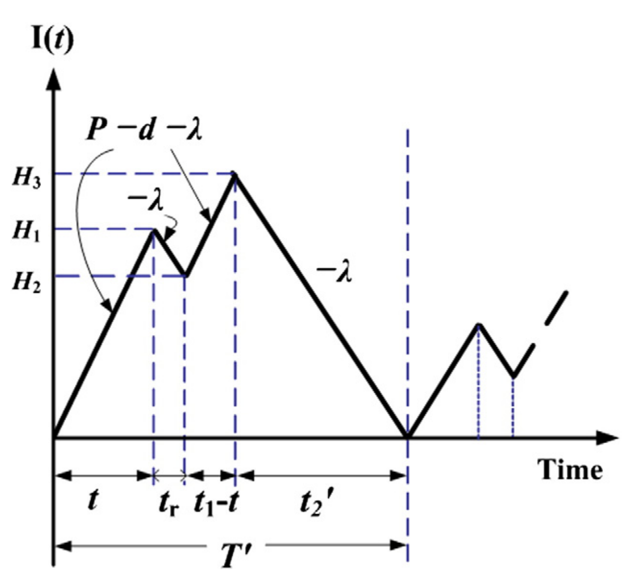

Fig. 1. On-hand inventory of perfect quality items in an EPQ-based model with nonconforming items and breakdown under abort/resume policy.