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An inventory model with time dependent demand and shortages under trade credit policy

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ABSTRACT

This model investigates an *economic order quantity (EOQ)* model over a finite time horizon for an item with a quadratic time dependent demand by considering shortages in inventory under permissible delay in payments. Shortages are assumed after variable time T_1(< T cycle period). The credit period (m) is known and fixed. The model is derived under three different circumstances depending on the time of occurrence of shortages, credit period, and cycle time. The results are illustrated with the help of numerical examples. The sensitivity analysis of key parameters of the optimal solution is studied with respect to changes in different parametric values. Some special features of the model are discussed.

1. Introduction

Since the formulation of EOQ in 1913, Harris's (1913) square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. This formula was developed on the assumption of constant demand. Thereafter many models were developed in the inventory literature by assuming constant demand. In the real marketing environment, the demand rate of any item may vary with time. Silver and Meal (1969) were the first to suggest a simple modification of the classical square root formula in the case of time varying demand. The classical no-shortage inventory problem for a linear trend in demand over a finite time horizon was analytically solved by Donaldson (1977). However, Donaldson (1977) solution's procedure was computationally complicated. Many researchers like Silver (1979), Ritchie (1985), Dave and Patel (1981), and Goyal (1986) made their valuable contributions in this direction. They did not consider shortages in their models. Deb and Chaudhuri (1987) were the first to extend the model of Silver (1979) to incorporate shortages in inventory. This extension was also studied by Dave (1989), Goyal et al. (1992), Goswami and Chaudhuri, (1991), Giri et al. (1996), and Teng (1996). Some researchers like Wee (1995) and Jalan and Chaudhuri (1999) developed their models by considering exponential time varying demand pattern. Sana and Chaudhuri (2000) extended the EOQ model over a finite time horizon by assuming unequal cycle lengths. From the existing literature, it is clear that while dealing with time varying demand, researchers have studied two types of demand rate, namely linear and exponential. A linearly time varying demand implies uniform change in demand rate of the product per unit time which is rarely seen to occur in real market. On the other hand, exponentially time varying demand indicates a very rapid change in demand which is also rare because the demand rate of any product cannot change with a high rate of change as exponential. Khanra and Chaudhuri (2003) was the first to consider a quadratic demand rate which is more realistic. Ghosh and Chaudhuri (2006) extended the EOQ model over a finite time horizon with shortages in all cycles.

In the conventional EOQ model, it was assumed that the customer must pay for the item as soon as it is received. In practice, however the supplier offers the retailer a certain trade credit period, in paying for purchasing cost. During this delay period, the retailer can earn revenue by selling items and by earning interest. An inventory model with permissible delay in payments was first studied by Goyal (1985). Several valuable contributions in this field were studied by Mondal and Phaujder (1989), Aggarwal and Jaggi (1995), Chu et al. (1998), Chung (2000) Sana and Chaudhuri (2008), Khanra et al. (2011), and Sarkar (2012a, 2012b,2013).

In this paper, an EOQ model is developed for an item with time varying quadratic demand and shortages and permissible delay in payments. However, this type of demand rate is more realistic because it can represent both accelerated growth and retarded growth in demand as it has the general form $D(t) = a + bt + ct^2$. Here $c = 0$ indicates linear time dependent demand and $a = 0$ as well as $b = 0$ simultaneously indicate constant demand. The model has been developed under three circumstances, Case 1: the credit period is less than the time of shortage period, Case 2: the credit period is greater than the time of commencement of shortage period but less than cycle length and Case 3: credit period is greater than the cycle for

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settling the account. The model is illustrated with numerical examples. Also, the sensitivity analysis of the model is examined for changes in parameters.

2. The mathematical model

The following assumptions are made to develop the model.

(i) The demand rate for an item is represented by a quadratic and continuous function of time.
(ii) Replenishment occurs instantaneously on ordering i.e.,, lead time is zero.
(iii) The inventory system involves only single type of item.
(iv) Shortages are allowed and completely backlogged.
(v) No interest is to be charged after commencement of shortages.
(vi) No interest is to be earned after permissible delay periods.
(vii) The planning period is of infinite length.

We consider the following notation to develop the model.

T	length of the replenishment cycle (decision variable) (year)
T_1	time when inventory level comes down to zero (decision variable) (year)
m	permissible delay in setting the account (year)
$I(t)$	inventory level at time t
k	ordering cost of inventory per order (\$/order)
h	unit holding cost per unit time excluding interest charges (\$/unit/unit time)
s	unit shortage cost per item (\$/item unit short)
p	unit purchase cost per item (\$/item)
$D(t)$	the time dependent demand rate (unit/year)

$$D(t) = a + bt + ct^2, a>0, b\neq 0, c\neq 0$$

Here a is the initial rate of demand and b is the rate with which the demand rate increases, the rate of change in the demand rate itself changes at a rate c.

I_e	the interest earned per year (\$/year)
I_r	the interest charges per year, $I_r \geq I_e$ (\$/year)

The inventory level $I(t)$ at time t generally decreases mainly to meet the demand only. Thus, the variation of inventory with respect to time can be described by the following differential equation

$$\frac{dI(t)}{dt} = -D(t); 0\leq t\leq T \tag{1}$$

with $I(T_1) = 0, I(0) = Q_0$.

The initial stock in any cycle is Q_0. The inventory decreases due to demand only. The inventory levels become zero at T_1. Shortages accumulate over $[T_1, T]$. The permissible delay period m is settled by the whole-seller or distributor to the retailer or customer. Thus, three models can be developed depending on different values of m, T_1 & T.

The solution of Eq. (1) is

$$I(t) = a(T_1 - t) + \frac{b}{2}\left(T_1{}^2 - t^2\right) + \frac{c}{3}\left(T_1{}^3 - t^3\right); \quad 0\leq t\leq T. \tag{2}$$

The initial order quantity is $I(0) = aT_1 + \frac{b}{2}T_1{}^2 + \frac{c}{3}T_1{}^3$.

In the interval $[0, T_1]$, the holding (HC) cost is

$$= h\int_0^{T_1} I(t)dt = h\left[\frac{a}{2}T_1{}^2 + \frac{b}{3}T_1{}^3 + \frac{c}{4}T_1{}^4\right]. \tag{3}$$

The shortage cost (SC) over the time interval $[T_1, T]$ is

$$= -s\int_{T_1}^{T} I(t)dt = s\left[\frac{a}{2}\left(T^2 - T_1^2\right) + \frac{b}{6}\left(T^3 - T_1^3\right) + \frac{c}{12}\left(T^4 - T_1^4\right) - \left(aT_1 + \frac{b}{2}T_1^2 + \frac{c}{3}T_1^3\right)(T - T_1)\right]. \tag{4}$$

Case 1. Let $m \leq T_1 < T$.

In Fig. 1, as the length of the period with positive stock is larger than the credit period, the buyer can use the sales revenue to earn interest at an annual rate I_e in $[0, T_1]$. The interest earned E_1 is given by

$$E_1 = pI_e\int_0^m tD(t)dt = pI_e\left(\frac{a}{2}m^2 + \frac{b}{3}m^3 + \frac{c}{4}m^4\right). \tag{5}$$

Moreover beyond the credit period, the unsold stock is supposed to be financed with an annual rate I_r and the interest payable I is given by

$$I = pI_r\int_m^{T_1} I(t)dt = pI_r\left[\frac{a}{2}T_1^2 + \frac{b}{3}T_1^3 + \frac{c}{4}T_1^4\right] - pI_r\left[\left(aT_1 + \frac{b}{2}T_1{}^2 + \frac{c}{3}T_1{}^3\right)m + \left(\frac{a}{2}m^2 + \frac{b}{6}m^3 + \frac{c}{12}m^4\right)\right]. \tag{6}$$

Thus, the total average cost per unit time is given by

$$\begin{aligned} Z_1(T_1, T) &= \frac{1}{T}[k + HC + SC + I - E_1] \\ &= \frac{1}{T}\Bigg[k + h\left(\frac{a}{2}T_1^2 + \frac{b}{3}T_1^3 + \frac{c}{4}T_1^4\right) + s\Bigg\{\frac{a}{2}\left(T^2 - T_1^2\right) + \frac{b}{6}\left(T^3 - T_1^3\right) \\ &\quad + \frac{c}{12}\left(T^4 - T_1^4\right) + \left(aT_1^2 + \frac{b}{2}T_1^3 + \frac{c}{3}T_1^4\right) - \left(aT_1T + \frac{b}{2}T_1^2T + \frac{c}{3}T_1^3T\right)\Bigg\} \\ &\quad + pI_r\left(\frac{a}{2}T_1^2 + \frac{b}{3}T_1^3 + \frac{c}{4}T_1^4\right) - pI_r\left(aT_1 + \frac{b}{2}T_1^2 + \frac{c}{3}T_1^3\right)m \\ &\quad + pI_r\left(\frac{a}{2}m^2 + \frac{b}{6}m^3 + \frac{c}{12}m^4\right) - pI_e\left(\frac{a}{2}m^2 + \frac{b}{3}m^3 + \frac{c}{4}m^4\right)\Bigg]. \end{aligned} \tag{7}$$

The EOQ in this case is $I(t) = aT_1 + \frac{b}{2}T_1{}^2 + \frac{c}{3}T_1{}^3$.

Our aim is to obtain the minimum average cost per unit time.

We first establish the following theorem to obtain minimum value of a function of two variables.

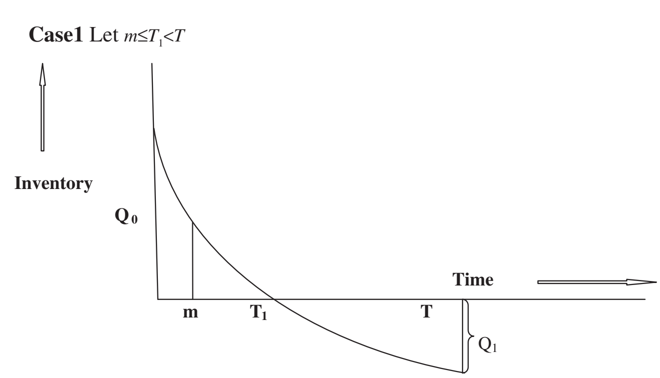

Fig. 1. Inventory versus time for Model I $m \leq T_1 < T$.