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The value of modeling with reference effects in stochastic inventory and pricing problems

M. Güray Güler *

Department of Industrial Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey

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ABSTRACT

We analyze a periodic review inventory system in which the random demand is contingent on the current price and the reference price. The reference price captures the price history and acts as a benchmark against which the current price is compared. The randomness is due to additive and multiplicative random terms. The objective is to maximize the discounted expected profit over the selling horizon by dynamically deciding on the optimal pricing and replenishment policy for each period. We study three key issues using numerical computation and simulation. First, we study the effects of reference price mechanism on the total expected profit. It is shown that high dependence on a good history increases the profit. Second, we investigate the value of dynamic programming and show that the firm that ignores the dynamic structure suffers from the revenue. Third, we analyze the value of estimating the correct demand model with reference effects. We observe that this value is significant when the inventory related costs are low.

1. Introduction

The reference price, or anchor price, is a benchmark which is developed by customers in repeated transactions. It is an internal standard against which prices are compared (Kalyanaram & Winer, 1995). The customers perceive the current price as *high* and the difference between the current price and the reference price as a *loss*, if the current price is greater than the reference price. Otherwise they perceive the current price as low and the difference as a gain. The reference price brings a challenge to companies. Although low prices may increase the current profit, they decrease the price expectations of the customers, therefore reduce the future profits. Hence there is a trade-off between current and future benefits in the presence of a reference price. In this study, we analyze joint pricing and inventory decisions when the random demand is subject to reference effects using simulation and computational analysis. Our aim is to find the effects of reference price mechanism on the optimal decisions and system profit. Moreover we investigate the value of modeling the problem with its underlying dynamic and reference effect structure.

Adaptation level theory, which states that expectation-based reference price is the adaptation level against which current prices are judged (Monroe, 1973), constitutes the theoretical basis of reference price (Helson, 1964). The reference price is dynamic in the sense that it evolves through time with the announcements of new price levels. Although there are different models used for the evolution mechanism (see e.g. Nasiry & Popescu, 2011), the exponential smoothing model which depends on the adaptive expectation model is the most commonly used form (Nerlove, 1958).

Fibich, Gavious, and Lowengart (2003) study dynamic pricing under deterministic linear demand with reference effects in a continuous time framework. They explicitly calculate the steady state prices and show that a firm should adopt a skimming or a penetration strategy depending on the initial reference price. Güler and Akan (2013) extend the study of Fibich et al. (2003) by incorporating an inventory decision and inventory related costs. They show that the cumulative structure of the holding cost yields an increase in the optimal prices. Popescu and Wu (2007) study the same problem for more general demand models when the time is discrete. They show that the skimming or penetration strategy can be generalized to non-linear models as well and explicitly calculate the steady state price levels. These papers study deterministic demand models.

Urban (2008) analyzes a single period model with random demand. He studies joint inventory-and-pricing model with both symmetric and asymmetric reference price effect, and provide numerical analysis which indicates that accounting for reference prices has a substantial impact on the firm's profitability.

Chen, Hu, Shum, and Zhang (2011) and Taudes and Rudloff (2012) study periodic review linear demand models with stochastic demand. Taudes and Rudloff (2012) study a single period and a two-period model. In the single period case, they show that the optimal inventory level and the optimal price increase in the

* Tel.: +90 462 325 6482.
E-mail addresses: mgguler@hotmail.com, guler@boun.edu.tr

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reference price. They prove the optimality of an state-dependent order-up-to (SDO) policy for the two-period case. In an SDO policy there is an optimal order-up-to level and price pair which depends on the state, i.e., the reference price. Chen et al. (2011) analyze finite and infinite horizon models and show the optimality of an SDO policy. They prove that the reference price converges to a steady state and provide characterizations of the steady state solution. In particular, they show that the optimal order-up-to level increases with the reference price. Güler, Bilgiç, and Güllü (2013a, 2013b) study concave demand models with stochastic demand. Güler et al. (2013a) show that the optimality of the SDO policy can be generalized to some concave demand models. Güler et al. (2013b) analyze models in which the randomness is due to an additive random term. They characterize the optimal pricing and inventory policy by showing that the problem can be decomposed into two subproblems and provide the solutions for the optimal parameters. These studies analyze analytical solutions for stochastic multi-period period problems with reference effects. Gimpl-Heersink, Rudloff, Fleischmann, and Taudes (2008) make a simulation of the multi-period problem with the linear demand where the randomness is due to an additive random term. They show that a base-stock list-price policy is optimal in the simulation. In such a policy, there is an optimal order-up-to level and a price pair at every period which are used if the on-hand inventory level is lower than the optimal order-up-to level, otherwise the firm does not order and goes to a discount. They also show that joint decision making for the inventory and the price brings a substantial increase in the profit for the demand models with reference effects.

For a detailed review on reference price, we refer the reader to Mazumdar, Raj, and Sinha (2005) and Arslan and Kachani (2010).

There is also a vast amount of literature on joint inventory and pricing without reference effects. Here we only give some of this literature and refer the reader to Elmaghraby and Keskinocak (2003) and Chen and Simchi-Levi (2012) for a detailed review of joint pricing and replenishment/inventory decisions. Federgruen and Heching (1999) study the periodic review multi-period problem where the unsatisfied orders are backordered. Chen and Simchi-Levi (2004a, 2004b) introduce a setup cost to the setting of Federgruen and Heching (1999). In these studies above, the optimal policy turns out to be a variant of an (s,S,p) policy. This policy states that if on hand inventory is below s, then the firm places an order to bring its inventory level to S such that $s \leqslant S$ and announces the price p. Otherwise it orders nothing and announces a state dependent price.

Although there are some analytical results for the joint inventory and pricing problem with reference effects under random demand, these results are limited due to the stochastic structure of the problem. The number of variables, together with stochasticity, increases the complexity of the problem. There are few studies which resort to numerical investigations to increase understanding of important and complex structure of the problem. Gimpl-Heersink (2008) provide computational studies for the linear model with an additive random term and show the effect of reference price on the optimal price and optimal inventory level. Gimpl-Heersink et al. (2008) show numerically the effects of different distributions for the additive random term. Both studies deal with the linear models. Güler et al. (2013a) provide some numerical illustrations for non-linear (concave) models which shows there is an evidence that the analytical results for the deterministic pricing problems hold for the stochastic problem as well. These two studies provide some insights for the problem; however there are quite a number of questions regarding the reference effects on pricing and inventory decisions. In this paper we provide a computational study and simulation to explore three research questions. First, we study the effects of reference price mechanism, i.e., the evolution mechanism and the initial reference price, on the total expected profit. Second, we investigate the value of dynamic programming. Third, we analyze the value of estimating the correct demand model with reference effects. For the last two goals, we compare two firms using Monte Carlo simulation. We use the demand models in Güler et al. (2013a) and use their optimality results in our computations.

The rest of this paper is organized as follows. We describe the demand model, formulate the problem with dynamic programming and set the values of the parameters in Section 2. The main analysis is given in Section 3. Finally, Section 4 concludes the paper and points to interesting topics for future research.

2. The model framework

In this section, we describe the demand model with reference effect, define the dynamic joint inventory and pricing model, and give the parameter values that we use throughout the analysis.

2.1. Description of the demand model with reference effects

We start by introducing the structure of the demand model and reference effect models that are used in this study. In stochastic inventory and pricing literature, the randomness of demand is generally modeled by incorporating an additive and/or a multiplicative random term over the mean demand which is a function of the price (p). In our setting, mean demand is also affected by the reference price (r). Hence the demand is determined by the following:

$$D_t(p,r,\theta_t,\epsilon_t) = \hat{d}_t(p,r)\theta_t + \epsilon_t.$$

Here $D_t(p,r,\theta_t,\epsilon_t)$ and $\hat{d}_t(p,r)$ are the demand and the mean demand functions of period t, respectively. Here θ_t and ϵ_t are independent random variables with $E[\theta_t] = 1$ and $E[\epsilon_t] = 0$ and they stand for the multiplicative and additive random terms, respectively. The variance of the demand is affected by the pricing decisions through the multiplicative term.

Reference effect on demand is defined as the excess or lost demand due to the change of the reference price at a given price level and is captured by the following functional relationship:

$$\mu_t(p) + R_t(r-p,r) = \hat{d}_t(p,r), \tag{1}$$

where $\mu_t(p) = \hat{d}_t(p,p)$ is called the base demand and $R_t(r-p,r)$ is called the reference effect on demand. In a demand model with reference effects, customers decide to buy the product comparing the reference price r and the price p. Hence the larger the difference, the greater the impact. Moreover the mean demand $\hat{d}(p,r)$ is assumed to be strictly decreasing in p and increasing in r (Chen et al., 2011; Gimpl-Heersink et al., 2008; Güler et al., 2013a, 2013b; Popescu & Wu, 2007). $R_t(r-p,r) = 0$ for $r = p$ by the definition of reference effect. Thus $R_t(r-p,r) \geqslant 0$ for $r > p$ and $R_t(r-p,r) \leqslant 0$ for $r < p$ since $R_t(\delta,r)$ is increasing with δ. Furthermore $R_t(r-p,r)$ is increasing with r and decreasing with p. For a detailed analysis on reference effect on demand, we refer the reader to Popescu and Wu (2007).

The general assumption on the bounds of the price and the reference price is that they are bounded with a lower and an upper bound, i.e., $p \in [\underline{p},\bar{p}]$ and $r \in [\underline{p},\bar{p}]$ where $\underline{p}$ and $\bar{p}$ are the lower and the upper bounds with $\underline{p} < \bar{p}$ respectively (Chen et al., 2011; Gimpl-Heersink et al., 2008; Popescu & Wu, 2007). Güler et al. (2013a), on the other hand, assume that the price and the mean demand has a lower bound. This lower bound on the mean demand imposes an upper bound on the price. We follow their definition and assume that the set of the price and the reference price is given by $\mathbb{P} = \{(p,r) : p \geqslant p^l,\ r \in [p^l,p^u],\ \hat{d}(p,r) \geqslant d^l\}$. Here p^l denotes the non-negative lower bound on the price and the reference price, p^u