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A deterministic inventory model for deteriorating items with two warehouses and trade credit in a supply chain system

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ABSTRACT

Chung and Huang (2007) designed recently a two-warehouse inventory model for deteriorating items when the supplier offers the retailer a delay period and in turn the retailer provides a delay period to their customers. They assumed that the stocks of RW are transported to OW via a continuous release pattern and the transportation costs are ignored. The holding cost in RW is exceeding that in OW. The deterioration rate of RW is assumed to be identical to that in OW. For practical purpose, it is observed that due to demand, the retailer needs to rent warehouse to store items sometimes. If the retailer's facility about deterioration is not newer than that of the rented warehouse, then $\alpha \geq \beta$. Otherwise, $\alpha < \beta$. This paper extends the model of Chung and Huang (2007) by considering $\beta > \alpha$ which means that the rate of deterioration in RW exceeds that of OW. First, expressions are obtained for the total variable cost of the inventory system. Second, this study demonstrates that a unique optimal solution exists. Third, two lemmas and one theorem are designed for determining the optimal cycle time. Finally, numerical examples are presented to illustrate the procedure for solving the model and sensitivity analysis of the optimal solution with respect to the parameters of the system is conducted.

1. Introduction

Basically, two-warehouse inventory systems have been considered by various researchers in the last few years. Such systems were first considered by Hartely (1976). Holding costs in the rented warehouse (RW) are assumed to exceed those in the owned warehouse (OW). Consequently, the items are stored first in OW and only excess stock is stored in the RW. Sarma (1983) designed a deterministic inventory model with two levels of storage and with an infinite replenishment rate. Murdeshwar and Sathe (1985) made an extension to the case of finite replenishment rate. Furthermore, Goswami and Chaudhuri (1992) developed a deterministic inventory model incorporating two levels of storage by considering linear demand trends. Zhou and Yang (2005) established a two-warehouse inventory model for items with stock-level-dependent demand rate and considering transportation cost. Additionally, the effect of deteriorating rate is vital in numerous inventory systems and cannot be ignored. Therefore, Sarma (1987) first presented a two-warehouse inventory model for deteriorating items with an infinite replenishment rate and allowing for shortages. Benkherout (1997) modified the model developed by Sarma (1987) by relaxing the assumptions of fixed cycle length and known quantity to be stocked in OW. Furthermore, Zhou (1998) developed a two-warehouse inventory model for deteriorating items under conditions of time-varying demand and shortages during the finite-planning horizon. Pakkala and Achary (1992a, 1992b) designed a two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages for the case of continuous and discrete release patterns in the rented warehouse, respectively. In their analysis, the transportation costs associated with transferring the items from the RW to the OW were not taken into account. Additionally, deterioration degree depends on the preservation of inventory in the facility, and thus on the environmental conditions in the warehouse. Therefore, all the above models have constant deterioration rates, with those in RW being less than those in OW. Hiroaki and Toyokazu (1996) explored perishable inventory control with two types of customers and different selling prices under the warehouse capacity constraint. Additionally, Bhunia and Maiti (1998) designed a two warehouse inventory model for deteriorating items with a linear trend in demand and shortages. The model assumed a positive deterioration rate in OW and a deterioration rate in RW of less than one. Later, Yang (2004) designed a two-warehouse inventory model for deteriorating items with shortages

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under inflation. The model assumes that the deterioration rate in RW exceeds that of OW and that both range between zero and one. Yang (2006) improved upon the model introduced in Yang (2004) to incorporate partial backlogging and relaxed the assumption that the deterioration rate in RW exceeds that of OW. Lee (2006) devised a two-warehouse inventory model with deterioration under FIFO dispatching policy. Lee (2006) assumed the deterioration rate in OW to be less than one and while that in RW is positive. Dye et al. (2007) developed a deterministic inventory model for deteriorating items under capacity constraints and a time-proportional backlogging rate. Yang (2012) explored two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. Wang et al. (2012) developed an inventory for a deteriorating item while the buyer has warehouse capacity constraint. Zhong and Zhou (2013) reveal that improving the supply chain's performance through trade credit under inventory-dependent demand and limited storage capacity. They assumed deterioration rates in RW and OW ranging between zero and one, respectively. The major assumptions used in the related previous articles are summarized in Table 1.

On the other hand, the influence of a permissible delay in payments on the optimal inventory system is an issue of consequence in practical environments. Therefore, numerous researchers have designed analytical inventory models that consider permissible delays in payments such as Haley and Higgins (1973), Goyal (1985), Arcelus and Srinivasan (1993), Shah (1993a, 1993b), Jaggi and Aggarwal (1994), Aggarwal and Jaggi (1995), Chung (1998), Jamal et al. (1997, 2000), Shinn (1997), Hwang and Shinn (1997), Sarker et al. (2001), Shinn and Hwang (2003), Huang and Liao (2008), Liao (2008a, 2008b), Liao and Chung (2009), Chung and Liao (2004, 2006, 2009, 2011), Thangam (2012) and their references.

Combining the above arguments, few inventory models with two-warehouses have been found in the literature that address the conditions associated with permissible delays in payments. Recently, Chung and Huang (2007) developed a two-warehouse inventory model for deteriorating items in which the supplier offers the retailer a permissible delay period and the retailer in turn provides a trade credit period to their customers. They assumed that stocks of RW are transported to OW using a continuous release pattern and transportation costs are ignored.

Table 1
Summary of related literatures for two-warehouse inventory model.

Author(s) and year	EOQ or EPQ	Deterioration rate in OW (α) and deterioration rate in RW (β)
Sarma (1983)	EOQ	$\alpha=\beta=0$
Murdeshwar and Sathe (1985)	EPQ	$\alpha=\beta=0$
Sarma (1987)	EOQ	$\alpha>\beta$
Goswami and Chaudhuri (1992)	EOQ	$\alpha=\beta=0$
Bhunia and Maiti (1994)	EOQ	$\alpha=\beta=0$
Benkherout (1997)	EOQ	$\alpha>\beta$
Bhunia and Maiti (1998)	EOQ	$0<\alpha;\beta<1$
Yang (2004)	EOQ	$\alpha<\beta;0<\alpha<1;0<\beta<1;$
Zhou and Yang (2005)	EOQ	$\alpha=\beta=0$
Yang (2006)	EOQ	$\alpha\neq\beta;0<\alpha<1;0<\beta<1$
Dye et al. (2007)	EOQ	$0\le\alpha<1;0\le\beta<1$
Lee (2006)	EOQ	$\alpha<1;\beta>0$
Chung and Huang (2007)	EOQ	$\alpha=\beta$
Hsieh et al. (2007)	EOQ	$0\le\alpha<1;0\le\beta<1$
Lee and Hsu (2009)	EPQ	$0<\alpha;\beta>0$
Liang and Zhou (2011)	EOQ	$\alpha>\beta$
Liao and Huang (2010)	EOQ	$\alpha<\beta$
Liao et al. (2012)	EOQ	$\alpha=\beta$
Liao et al. (2s013)	EOQ	$\alpha=\beta$
Present paper	EOQ	$\alpha<\beta$

Both RW and OW had identical rates of deterioration and that the holding cost for RW exceeds that in OW. In real life situation, owing to demand, the retailer needs to rent warehouse to store items sometimes. If the retailer's facility about deterioration is not newer than that of the rented warehouse, then $\alpha\ge\beta$. Otherwise, $\alpha<\beta$. This paper extends the model Chung and Huang (2007) by assuming $\beta>\alpha$. That is, the deterioration rate in RW exceeds that of OW. Firstly, expressions are derived for the total variable cost of the inventory system, respectively. Secondly, this study shows that the optimal solution not only exists but also is unique. Thirdly, five lemmas and two theorems are developed for optimizing the optimal cycle time. Finally, numerical examples demonstrating the applicability of the proposed model and conducting sensitivity analysis on the model parameters are also discussed.

2. Model formulation

The following notations and assumptions will be used throughout the whole paper.Notations

OW	the owned warehouse
RW	the rented warehouse
D	the demand rate per unit time
A	the replenishment cost per order
W	the storage capacity of the owned warehouse
T	the length of replenishment cycle
Q	the order quantity per replenishment
s	the selling price per unit item
c	the purchasing cost per unit item
h_0	the holding cost per unit per unit time in OW
h_r	the holding cost per unit per unit time in RW and $h_r\ge h_0$
α	the deterioration rate in OW, where $0<\alpha$
β	the deterioration rate in RW, where $0<\beta$ and $\beta>\alpha$
T_w	the time at which the inventory level reaches zero in RW
$I_0(t)$	the inventory level in OW at time t
$I_R(t)$	the inventory level in RW at time t
$I_{01}(t)$	the level of inventory at OW during the time interval $(0,T_w)$
$I_{02}(t)$	the level of inventory at OW during the time interval (T_w,T)
T_w	the time at which the inventory level reaches zero in RW
T_a	$\frac{1}{\alpha}\ln(1+\frac{\alpha W}{D})$
M	permissible delay in settling the accounts
I_p	the interest charged per dollar in stocks per year
I_e	the interest earned per dollar per year

Assumptions:

(1) Demand rate is known and constant.
(2) Shortages are not allowed.
(3) The time horizon is infinite.
(4) Replenishment rate is infinite and lead time is zero.
(5) The owned warehouse (OW) has a fixed capacity of W units.
(6) The rented warehouse (RW) has unlimited capacity.
(7) The items of RW are consumed first and next the items of OW.
(8) When $T\ge M$, the account is settled at $T=M$. Beyond the fixed credit period, the retailer begins paying the interest charges on the items in stock at rate I_p. Before the settlement of the replenishment account, the retailer can use the sale revenue to earn the interest at annual rate I_e, where $I_p\ge I_e$.
(9) When $T\le M$, the account is settled at $T=M$ and the retailer does not need to pay any interest charge. Alternatively, the retailer can accumulate revenue and earn interest until the end of the trade credit period.