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Joint pricing and ordering policy for two echelon imperfect production inventory model with two cycles

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ABSTRACT

This study deals with an imperfect EPQ (economic production quantity) price dependent inventory model over two types of cycles: in the first cycle, the retailer sells only good product with actual price and, in the second, he sells the products with a discount price. In the production run-time, the non-conforming items are produced at a random rate and they are reworked after the regular production run time and the reworked items are almost perfect as good as original quality items. The retailer starts the second cycle when a certain percent of good items are left to him, after the completion of regular production. The retailer continues simultaneously two cycles up to which both types of the products are available to him. Now, our objective is to find out optimal ordering lot size and optimal price for which the profit of the integrated model is maximum. Also, we check the best strategy of retailer for the time of starting second cycle. A numerical example is provided to illustrate the behavior and application of the model.

1. Introduction

Inventory management is an important part of a business because it ensures quality control in the businesses and handles transactions revolving around consumer goods. Without proper inventory control, a large retail store may run out of stock on an important item or may stock out at an important time. The business industries basically use an inventory management system that will trace and maintain the requirement of inventory to meet customers' demand. Pricing of commodities is a fundamental element of the inventory management to determine profits actively and find out the demand rate of products, ordering lot size of the company, etc. A pricing strategy helps one business company to meet his sales objectives, enhances his reputation and provides the best policy for market demand.

The basic traditional inventory control model was first studied by Harris (1913a, 1913b, 1915). He introduced an economic order quantity (EOQ) model which indicates a firm how much should be ordered and when orders should take place so that the sum of the inventory costs will be minimized. After a couple of decades, Wilson (1934) applied and analyzed extensively the Harris model. Besides many authors like Arrow et al. (1951), Whitin (1954) and Erlenkotter (1990) analyzed and reviewed Harris' model. Abad (1988) studied a joint price and lot-size determination problem where the supplier offered incremental quantity discounts to the retailer on purchasing of products. Wee (1995) developed a deterministic joint pricing and replenishment model for constant deteriorating items when the demand rate was exponentially decreasing with time. Wee and Yu (1997) analyzed an inventory system of a deteriorating commodity having a temporary price discount period. A modified inventory model for imperfect quality items using the EPQ/EOQ formulae was formulated by Salameh and Jaber (2000). They showed that the economic lot size quantity increases when the average percentage of imperfect quality items increases. Huang (2002) developed a model to determine an optimal integrated vendor–buyer inventory policy for flawed items in a just-in-time (JIT) manufacturing environment. Viswanathan and Wang (2003) introduced the effectiveness of quantity discounts and volume discounts coordination mechanisms with price-sensitive demand in a simplified setting of a single-vendor and single-retailer in a distribution channel. Yang (2004) evaluated an optimal replenishment and pricing policy for price sensitive demand. Jamal et al. (2004) studied an imperfect production inventory model to obtain an optimal production lot size with provision of reworking the defective items. They formulated the model under two different operational policies, reworking are completed within the same cycle and reworking are done after N cycles, to minimize the total system cost. Wang (2004) introduced an imperfect economic production inventory model where the products were sold under a free-repair warranty policy. Teng and Chang (2005) developed an economic production quantity (EPQ)

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model for deteriorating items when the demand rate depended not only on the level of stock-display level but also on the selling price per unit. You (2005) investigated the problem of joint determination of order size and optimal prices for a perishable inventory system under the condition that demand is time and price dependent. Sarker and Kindi (2006) introduced an EOQ model with a discounted price that determines the optimal ordering policy during the sale period for different scenarios. The correct derivation of the optimal ordering policies, the correct mathematical expressions and the appropriate solution to the numerical examples of the model of Sarker and Kindi (2006) were modified by Cardenas-Barron (2009a). Wee et al. (2007) studied an optimal inventory model for the items with imperfect quality and shortage backordering. Maddah and Jaber (2008) modified the model of Salameh and Jaber (2000) with unreliable supply, characterized by a random fraction of imperfect quality items and a screening process. Cardenas-Barron (2009b) also extended the model of Sarker and Kindi (2006) where he introduced a simple alternative optimization technique based on the concept of parabola from analytic geometry. Sana (2011a) introduced integrated three layer production–inventory model for imperfect manufacturing system where order size of raw materials and production rate were decision variables. Pal et al. (2012a) studied a deterministic multi-items inventory model considering price and level of price breaks dependent demand rate. Glock (2012) reviewed a lot-size model which was highlighted on coordinated inventory replenishment decisions between buyer and vendor and their impact on the performance of the supply chain. Many researchers (Pal et al., 2012b, 2013; Cardenas-Barron, 2006, 2008, 2009c, 2012; Goyal and Cardenas-Barron, 2002; Sarkar et al., 2010, 2011; Sarkar and Moon, 2011; Hayek and Salameh, 2001; Lin et al., 2003; Papachristos and Skouri, 2003; Chung et al., 2009; Wee and Widyadana, 2013; Rezaei and Salimi, 2012; Hsu and Hsu, 2013; Dhouib et al., 2012; Papachristos and Konstantaras, 2006; Konstantaras et al., 2007; Lo et al., 2007, Král, 2003; Sana, 2010, etc.) also studied imperfect production inventory model, considering various issues. Pricing in a inventory model has frequently been studied in the past (see You and Hsieh, 2007; Sana, 2011b, 2011c; Choi, 1991, etc.)

In this paper, we introduce a two echelon production inventory model over two cycles involving single supplier and single retailer. The existing production system is not perfect and it produces non-conforming products at a random rate which follows a probability distribution. All the defective items are remanufactured after the regular production run-time and these are almost perfect as good as original products. The retailer sells the good products with actual price in the first cycle and the remanufactured products are sold with discount price in the second cycle. Now, the objective of our problem is to find out the optimal inventory lot-size and optimal selling price so that the total expected profit per unit quantity of the chain is maximized.

The model can be useful for the industries like textile, footwear, and electronics. When retailer produces these products, some defective products are also produced along with good products. These defective items are remanufactured or repaired to sell in the markets. At the end of a season, companies generally offer discount to the customers to finish stock. Also, some times, companies offer discount to boost the items which are almost perfect but not fully. Finally, we formulate the expected profit functions per unit quantity of the members of the chain.

The rest of the paper is organized as follows: Section 2 illustrates fundamental assumptions and notations. Formulation of the model is discussed in Section 3. Section 4 analyzes numerical analysis. Sensitivity analysis is illustrated in Section 5 and finally conclusion of the paper is provided in Section 6.

2. Fundamental assumptions and notation

2.1. Assumptions

The following assumptions are adopted to develop the model:

(i) Model is developed for single item over two cycles: in the first cycle, retailer sells product with actual price and in the second cycle, the retailer sells products with special discount on selling price.
(ii) Replenishment rate of supplier is instantaneously infinite, but its size is finite, that means replacement of lot size is sufficiently large at any number, if needed.
(iii) Production rate of the retailer is constant and is greater than the constant part of the demand rate of the customers.
(iv) Production rate of defective items is random and no scrap item is produced during production time.
(v) In each production run time, reworking starts just after the end of regular production process and the reworked items are almost perfect as good as original quality products. Reworking rate of the retailer is constant and is greater than production rate.
(vi) Inventory holding cost of good, defective and reworked items is different.
(vii) Stock-out situation in each stage is not allowed.

2.2. Notations

The following notations are used throughout the paper:

Q	replenishment lot size (unit) of supplier
$D(p)$	demand rate of the customer
p	selling price ($/unit) for the retailer
$D(0)$	production rate of good items of the retailer
$I_s(t)$	on-hand inventory of raw material at time t for the supplier
I_{m_g}	on-hand inventory of good products at time t for the retailer
I_{m_d}	on-hand inventory of defective products at time t for the retailer
$\alpha D(0)$	production rate of defective items of the retailer where α is a random variable
P_r	remanufacturing rate of defective items of the retailer
C_{hs}	holding cost ($) per unit per unit time for the supplier