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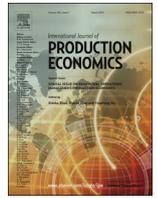
An inventory model with both stock-dependent demand rate and stock-dependent holding cost rate

Chih-Te Yang *

Department of Industrial Management, Chien Hsin University of Science and Technology, Taoyuan 320, Taiwan

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ABSTRACT

In this paper, we develop an inventory model under a stock-dependent demand rate and stock-dependent holding cost rate with relaxed terminal conditions. Shortages are allowed and partially backlogged in the model. The purpose of this study is to determine the optimal order quantity and the ending inventory level such that the total profit per unit time is maximized for the retailer. We first establish a proper model for a mathematical formulation. Then we develop several theoretical results and provide the decision-maker with an algorithm to determine the optimal solution. Finally, numerical examples are provided to illustrate the solution procedure, and a sensitivity analysis of the optimal solution with respect to major parameters is carried out.

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1. Introduction

Since Harris (1913) presented an economic order quantity (EOQ) model, many researchers have been made to adjust their assumptions to more realistic situations in inventory management. For instance, it is usually observed in the supermarket that the display of such goods in large quantities attracts more customers and generates a higher demand (Levin et al., 1972). The inventory problem is an issue that has received considerable attention in inventory models with stock-dependent demand rates. Gupta and Vrat (1986) were the first to develop an inventory model for stock-dependent consumption rates. Later, Baker and Urban (1988) also developed an EOQ model for a power-form stock-dependent demand pattern. Mandal and Phaujdar (1989) then proposed an economic production quantity (EPQ) model for deteriorating items based on a constant production rate and linearly stock-dependent demand. Datta and Pal (1990) presented an inventory model where the demand rate is a piecewise function of the inventory level. Pal et al. (1993) extended Baker and Urban's (1988) model for deteriorating items. Padmanabhan and Vrat (1995) presented a deteriorating inventory model based on a stock-dependent selling rate and shortage. Wu et al. (2006) developed a replenishment model for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Examples of other studies in this area include Sarker et al. (1997), Ray and Chaudhuri (1997), Ray et al. (1998), Dye and Ouyang (2005), Lee and Dye (2012), Min et al. (2012), Avinadav et al. (2013) and Taleizadeh et al. (2013).

Furthermore, several inventory models assume that the holding cost per unit time is constant. In real life, the holding cost for perishable goods such as foodstuffs, milk, fruit, vegetables, and meat drops with each passing day, and increasing holding costs are necessary to maintain the freshness of the items and to prevent spoilage. Weiss (1982) supposed that the holding cost per item is a convex potential function of time. Goh (1994) extended Baker and Urban's (1988) model to relax the assumption of a constant holding cost. Later, Giri and Chaudhuri (1998) extended Goh's (1994) model and developed an inventory model for deteriorating items. Recently, Pando et al. (2012) formulated an inventory model with both the demand rate and holding cost dependent on the stock level. Other studies in this area include those of Hwang and Hahn (2000), Alfares (2007), Roy (2008), Valliathal and Uthayakumar (2011), Pando et al. (2013), and Tripathi (2013).

One of the major assumptions used in the above models is that the replenishment cycle would end with zero stock. However, in real life it may be desirable to order larger quantities, resulting in stock remaining at the end of the cycle, due to the potential profits from the increased demand. Urban (1992) first relaxed the terminal condition of zero ending inventory and suggested that it is more profitable to utilize higher inventory levels resulting in greater demand. Mandal and Maiti (1999) formulated an EPQ model with a stock-dependent consumption rate for damageable items and some units in hand. Furthermore, Giri et al. (1996) extended the model of Urban (1992) for inventory items deteriorating at a constant rate. Chang (2004) amended Giri and Chaudhuri's (1998) model for deteriorating items by changing the objective to the maximization of profit and relaxing the restriction of a zero ending inventory. Teng et al. (2005) extended Urban's (1992) model to accommodate not only deteriorating but also non-zero ending inventory, and proposed an algorithm to

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* Tel.: +886 3 4581196x6123; fax: +886 3 250 3908.
E-mail address: ctyang@uch.edu.tw

obtain the optimal replenishment cycle time and ordering quantity. Recently, Chang et al. (2010) extended Wu et al. (2006) model to relax the restriction of zero ending inventory when shortages are not desirable. The major assumptions used in the above-mentioned studies are summarized in Table 1.

Thus, the problem of determining the optimal replenishment policy with a stock-dependent demand and stock-dependent holding cost rate is addressed in this paper in a manner that reflects realistic circumstances. The terminal condition of a zero ending inventory level is relaxed and shortages are allowed and partially backlogged. The rest of the paper is organized as follows. In the next section, we describe the notations and assumptions used throughout this paper. In Section 3, we establish the mathematical model. We then develop several theoretical results in Section 4 and provide the decision-maker with an algorithm for finding the optimal solution. A couple of numerical examples are provided in Section 5 to illustrate the solution procedure. In addition, a sensitivity analysis of the optimal solution with respect

Table 1
Major characteristics of inventory models on selected researches.

Literature	EOQ/EPQ model	Demand rate	Holding cost	Deterioration	Non-zero ending inventory	Shortage
Harris (1913)	EOQ	Constant	Constant	No	No	No
Weiss (1982)	EOQ	Constant	Nonlinear time-dependent	No	No	No
Baker and Urban (1988)	EOQ	Stock-dependent (power function)	Constant	No	No	No
Mandal and Phaujdar (1989)	EPQ	Stock-dependent (linear function)	Constant	Constant	No	No
Datta and Pal (1990)	EOQ	Stock-dependent (piecewise function)	Constant	Constant	No	No
Urban (1992)	EOQ	Stock-dependent (piecewise function)	Constant	No	Yes	No
Pal et al. (1993)	EOQ	Stock-dependent (power function)	Constant	Constant	No	No
Goh (1994)	EOQ	Stock-dependent (power function)	Nonlinear stock dependent	No	No	No
Padmanabhan and Vrat (1995)	EOQ	Stock-dependent (linear function)	Constant	Constant	No	Completely/partially backlogged
Giri et al. (1996)	EOQ	Stock-dependent (power function)	Constant	Constant	Yes	No
Ray and Chaudhuri (1997)	EOQ	Stock-dependent (power function)	Constant	No	No	Completely backlogged
Sarker et al. (1997)	EPQ	Stock-dependent (linear function)	Constant	Constant	No	Completely backlogged
Giri and Chaudhuri (1998)	EOQ	Stock-dependent (power function)	Nonlinear time/stock-dependent	Constant	No	No
Ray et al. (1998)	EOQ	Stock-dependent (power function)	Constant (two warehouses)	No	No	No
Mandal and Maiti (1999)	EPQ	Stock-dependent (power function)	Constant	Stock-dependent	Yes	No
Hwang and Hahn (2000)	EOQ	Stock-dependent (power function)	Constant	Fixed lifetime	Yes	No
Chang (2004)	EOQ	Stock-dependent (power function)	Nonlinear time/stock-dependent	Constant	Yes	No
Dye and Ouyang (2005)	EOQ	Stock-dependent (linear function)	Constant	Constant	No	Partially backlogged
Teng et al. (2005)	EOQ	Stock-dependent (piecewise function)	Constant	Constant	Yes	No
Wu et al. (2006)	EOQ	Stock-dependent (linear function)	Constant	Non-instantaneous	No	Partially backlogged
Alfares (2007)	EOQ	Stock-dependent (linear function)	Nonlinear time-dependent	No	No	No
Roy (2008)	EOQ	Price-dependent (linear function)	Linear time-dependent	Time-dependent	No	Completely backlogged
Chang et al. (2010)	EOQ	Stock-dependent (linear function)	Constant	Non-instantaneous	Yes	Partially backlogged
Valliathal and Uthayakumar (2011)	EPQ	Stock and time-dependent	Nonlinear stock-dependent	Time-dependent	No	Partially backlogged
Pando et al. (2012)	EOQ	Stock-dependent (power function)	Nonlinear stock-dependent	No	No	No
Lee and Dye (2012)	EOQ	Stock-dependent (linear function)	No	Controllable	No	Partially backlogged
Min et al. (2012)	EPQ	Stock-dependent (linear function)	Constant (excluding interest charges)	No	No	No
Avinadav et al. (2013)	EOQ	Price and Time-depend (power function)	Constant	Constant	No	No
Pando et al. (2013)	EOQ	Stock-dependent (power function)	Nonlinear stock and time –dependent	No	No	No
Taleizadeh et al. (2013)	EOQ	Constant	Constant	Constant	No	Completely backlogged
Tripathi (2013)	EOQ	Time-dependent (power function)	Linear time-dependent	No	No	No
This paper	**EOQ**	**Stock-dependent (power function)**	**Nonlinear stock-dependent**	**No**	**Yes**	**Partially backlogged**