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An inventory model for deteriorating items with shortages and time-varying demand

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ABSTRACT

A deterministic inventory system for items with a constant deterioration rate is studied. Demand varies in time and it is assumed that it follows a power pattern. Shortages are allowed and backlogged. The ordering cost, the holding cost, the backlogging cost, the deteriorating cost, and the purchasing cost are considered in the inventory management. An approach is proposed to minimize the total cost per inventory cycle. This cost depends on two decision variables: the time at which the inventory level falls to zero and the length of the scheduling period. Numerical examples illustrate the theoretical results.

1. Introduction

Management of inventories represents one of the most important areas in business, trade and industry. An inventory problem consists of making optimal decisions to minimize the total cost of the inventory system. In general, the decisions pertaining to an inventory problem deal with two questions: when the inventory should be replenished and how much should be added to the inventory. Two elements have contributed to the wide range of complexity in the inventory models: the nature or form of demand during the scheduling period of the inventory and the possibility or not of incurring shortages in the inventory. In the inventory systems, shortages arise due to lack of stock to supply customer demand. Combinations of these elements or properties result in a spectrum of inventory models with different applications. Three kinds of costs are significant in the management of inventories: the cost of carrying inventories, the cost of incurring shortages and the cost of replenishing inventories. Usually, it is assumed that shortages are backlogged, i.e., shortages accumulated during a period can be made up immediately after replenishment has occurred at the beginning of the next inventory cycle. Logically, it has a shortage cost known as backlogging cost. Taking into account the properties and assumptions that characterize the inventory system, the decisions on the inventory management are usually made as a function of variables that are subject to control in the system. These controllable variables affect the inventory costs and the problem consists of finding their specific values that minimize the total inventory cost.

In the basic inventory models, such as the economic order quantity (EOQ) model proposed by Harris (1913), a frequent assumption consists in supposing that demand is constant. However, inventory models must consider that, in practice, customer's demand changes with time. Thus, the study of the inventory systems with time-varying demand is interesting because it allows to appropriately modelling the behavior and the evolution of the inventory. Many researchers have analyzed inventory models where the demand rate varies with time. Thus, Silver and Meal (1973) studied an approximate approach for a deterministic time-varying demand pattern. Donaldson (1977) analyzed the classical no-shortage inventory policy for a linear trend in demand. Siver (1979) proposed an inventory replenishment decision rule for a linear demand. Ritchie (1984) studied the solution procedure for an inventory model with linear increasing demand. Goswami and Chaudhuri (1991) developed an economic order quantity (EOQ) model, allowing shortages and with a linear trend in demand. Teng (1996) analyzed a deterministic inventory replenishment model with a linear trend in demand. Goyal and Giri (2003) proposed a rule for determining replenishment periods of an inventory with linear decreasing demand rate. Zhao et al. (2001) analyzed heuristics for inventory replenishment with linear decreasing demand. Wen-Yang et al. (2002) developed an optimal

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inventory replenishment policy for a linear trend in demand. Yang et al. (2004) studied an approach to analyze an inventory system with non-linear decreasing demand. Sakaguchi (2009) developed an inventory policy for a system with time-varying demand. Omar and Yeo (2009) presented an inventory model that satisfied a continuous time-varying demand.

In daily life, the deteriorating of goods is a frequent and common phenomenon. Fruits, foods, vegetables, cakes, sweets, and pharmaceuticals are a few examples of such goods. A deterioration process is defined as damage, decay, obsolescence and loss of value in a product along time. In the stocked products it is usual that a percentage of the items can undergo deterioration or decay. This results in the decreasing usefulness and operation of the product. Commodities such as vegetables, fruits, yogurts, milk, fish, meats, drug, etc. have a fixed and short period of lifetime. These commodities are referred to as deteriorating items.

Many researchers have engaged their attention to study inventory models for deteriorating items. Thus, Hadley and Whitin (1963) presented a study of fashion goods deteriorating at the end of the storage period. Ghare and Schrader (1963) analyzed an inventory model for goods with an exponential decay. Misra (1975) developed a production lot-size model for an inventory system with deteriorating items. Shah and Jaiswal (1977) carried out an order-level inventory model considering a constant deterioration rate. Aggarwal (1978) analyzed the model proposed by Shah and Jaiswal, rectifying an error in calculating the average inventory holding cost. Dave and Patel (1981) presented an inventory model with deterioration, time-proportional demand and instantaneous replenishment. Chowdhury and Chaudhuri (1983) studied an order-level system for deteriorating items assuming finite rates of replenishment and shortage. Mitra et al. (1984) analyzed deteriorating order quantities assuming a linear trend in demand. Bahari-Kashani (1989) studied the optimal replenishment for deteriorating items with time-proportional demand. Raafat (1991) presented a survey of literature on deteriorating inventory models. Bose et al. (1995) analyzed an EOQ model for deteriorating items with a linear positive trend in demand and shortages backlogged. Chakrabarti and Chaudhuri (1997) developed an EOQ model for deteriorating items with shortages and a linear trend in demand. Chang and Dye (1999) discussed an EOQ model for deteriorating items with time varying demand and partial backlogging. Goyal and Giri (2001) discussed a detailed review of deteriorating inventory literatures. Wu (2002) also discussed an EOQ model with time-varying demand, considering deterioration and shortages. Li et al. (2010) analyzed a review on inventory management for deteriorating items. Mishra and Singh (2010) developed an inventory model with constant rate of deterioration and time dependent demand. Yan et al. (2011) developed an integrated production–inventory model for a deteriorating inventory item. Balkhi (2011) presented an optimal ordering policy for deteriorating items under different supplier trade credits and considering a finite horizon. Yang (2012) discussed two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. Musa and Sani (2012) studied inventory ordering policies for deteriorating items under permissible delay in payments. Taleizadeh et al. (2013) analyzed an EOQ model for perishable product with special sale and assuming shortages.

The demand patterns are referred to as different ways by which products are taken out of inventory during the scheduling period to supply customer demand. If the demand rate is the same during all the inventory cycles, the demand pattern is known as uniform demand pattern. However, there are other ways by which the units may be withdrawn throughout the period. The power demand pattern allows suiting the demand to more practical situations. Thus, this pattern allows representing the behavior of demand when it is uniformly distributed throughout the period, and also modelling situations where a high percentage of units may be mainly withdrawn either at the beginning or at the end of the period.

Several papers on inventory systems consider that the demand follows a power pattern. Thus, Goel and Aggarwal (1981) studied an order-level inventory system with power demand pattern for deteriorating items. Datta and Pal (1988) analyzed an inventory model with power demand pattern and variable rate of deterioration. Lee and Wu (2002) presented an inventory model for items with deterioration, shortages and power demand pattern. Dye (2004) extended this last model to a general class with time-proportional backlogging rate. Singh et al. (2009) analyzed an EOQ model for perishable items with power demand pattern and partial backlogging. Rajeswari and Vanjikkodi (2011) studied an inventory model for deteriorating items with partial backlogging and power demand pattern. Recently, Mishra and Singh (2013) presented an economic order quantity model for deteriorating items with power demand pattern and shortages partially backlogged.

In the inventory models with a power demand pattern cited in the last paragraph, the length of the inventory cycle is fixed and known. In those models the variable to determine is usually the time at which the inventory level reaches zero. Thus, the costs associated with the inventory management depend on this unique variable. Hence, the average total cost per inventory cycle is also a function that depends only on this variable. Also, the papers of Rajeswari and Vanjikkodi (2011) and Mishra and Singh (2013) considered in their formulations only approximations of the real inventory cost function. Thus, their results give an approximate inventory policy when the inventory cycle is fixed, but they do not determine the inventory policy when the inventory cycle is unknown.

However, in this paper, the scheduling period or inventory cycle is not fixed and the average total cost per inventory cycle depends on two decision variables: the time at which the inventory level reaches zero and the length of the scheduling period. Moreover, we present the exact formulation of the inventory cost function without approximations. It implies that this new model better represents the real inventory problem and its solution requires a major effort in determining the efficient inventory management policy.

In this paper we analyze a deterministic inventory model assuming that demand follows a power demand pattern, shortages are backlogged and the products have a deterioration process. The organization of the paper is as follows. In Section 2 we introduce the notation used throughout the paper and the basic assumptions of the inventory system. In the third section we develop the mathematical model that describes the evolution of the inventory system. In Section 4 a procedure to solve the inventory problem is presented, determining the condition that must be satisfied by the inventory policy. Numerical examples are provided in Section 5 to illustrate the solution procedure. Section 6 presents a sensitivity analysis of the inventory policy. Lastly, conclusions and some themes on future studies are presented.

2. Notation and assumptions

We will use throughout the paper the following notation.

T Length of inventory cycle or scheduling period (time).
Q Lot size or order quantity (units).
S Initial stock level (units).
s Reorder point (units).
$I(t)$ Inventory level at time t.
t_0 Time at which the inventory level falls to zero.
A Ordering cost ($/order).
h Holding cost per unit and per time unit ($/unit and time unit).

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